129. *On the Spectral Decomposition of Dissipative Operators*

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An operator A on a Hilbert space is called *dissipative* if its imaginary part is non-negative, i.e.,

$$Im(A)=\frac{1}{2i}(A-A^*)\geqq 0.$$

In the present paper, we shall concern ourselves with the spectral properties of dissipative operators with completely continuous imaginary part, and deduce the spectral decomposition of operators of this class in the case of real spectrum. Consequently, for completely continuous dissipative operators with real spectrum, the canonical reduction of Jordan type will be established.

For the sake of simplicity, we shall assume that our (complex) Hilbert space is separable. By an operator we always understand a bounded linear transformation on a Hilbert space. Let A be an operator on a Hilbert space H. Then we denote by $\sigma(A)$ the spectrum of A and by $P_\sigma(A)$ (resp. $C_\sigma(A)$, $R_\sigma(A)$) the point (resp. continuous, residual) spectrum of A. If $\sigma(A)=P_\sigma(A)$ (resp. $\sigma(A)=C_\sigma(A)$), we say that A has a pure point (resp. continuous) spectrum.

As in [5]-[6], throughout this paper we make use of the concept of von Neumann algebra. Let us recall the terminologies and the notations used in [5]-[6]. An operator A is said to be *primary* if the von Neumann algebra $R(A)$ generated by A is a factor (that is, its center consists of scalar multiples of the identity operator I), and a primary operator A is said to be *of type* I_n (resp. *of type* I_∞) if a factor $R(A)$ is of type I_n (resp. of type I_∞).

1. A typical example of completely continuous dissipative operators is the integral operator A of Volterra type on $L_2(0, 1)$ defined by

$$(Af)(x)=i\int_0^x f(t)dt.$$

It is well known that A is a primary operator with non-hermitian rank 1 and $\sigma(A)=C_\sigma(A)=\{0\}$, that is, A has a pure continuous spectrum. In [5]-[6], we have shown that a non-scalar primary operator with non-hermitian rank 1 has, in general, a pure continuous spectrum in case $\sigma(A)$ is real. This spectral property may be generalized to the dissipative case.