125. *On Cauchy's Problem for a Linear System of Partial Differential Equations of First Order*

By Minoru YAMAMOTO

Osaka University

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1. **Introduction.** In this note we shall show the existence and the uniqueness of the solution for a linear system of partial differential equations of the following form (1.1) satisfying the prescribed initial conditions (1.2):

$$\frac{\partial u_\mu}{\partial t}=\sum_{\nu=1}^{k}\left\{\sum_{j=1}^{m}A_{\mu\nu j}(t,x)\frac{\partial u_\nu}{\partial x_j}+B_{\mu\nu}(t,x)u_\nu\right\}+f_\mu(t,x) \tag{1.1}$$

$$u_\mu(0,x)=\varphi_\mu(x)\qquad(\mu=1,2,\cdots,k) \tag{1.2}$$

under some conditions on $A_{\mu\nu j}$, $B_{\mu\nu}$, f_μ, and φ_μ which should be specified later (see [2]). We shall summarize here some notations and definitions. R^m denotes the m-dimensional Euclidean space whose elements are denoted by $x=(x_1, x_2, \cdots, x_m)$, and $z=x+iy=(x_1+iy_1, x_2+iy_2, \cdots, x_m+iy_m)$ $(x, y\in R^m, i=\sqrt{-1})$ is an element of m-dimensional complex space C^m. For some positive T, $D(T)=\{(t,x); 0\leqq t\leqq T, x\in R^m\}$ and $\mathfrak{D}_\gamma(T)=\{(t,z); 0\leqq t\leqq T, z=x+iy\in C^m, |y_j|<\gamma, j=1,2,\cdots, m\}$ for some positive γ.

A function $f(t,x)$ which is h-time continuously differentiable with respect to (t,x), is denoted by $f(t,x)\in C^h_{(t,x)}$, and that $f(t,x)$ which is analytic with respect to x for each $t\in[0,T]$ is denoted by $f(t,x)\in A_{(x)}$.

For any positive constants a and b, a function $f(t,x)$ belonging to $C_{(t,x)}$ on $D(T)$ and satisfying the inequality: $|f(t,x)|=Me^{ae^{b|x|}}$ on $D(T)$ for some positive constant M, is denoted by $f(t,x)\in F(a,b)$.

The method of the proof of the existence of the solution is essentially based on that of Prof. M. Nagumo [2]. The author wishes to express his deepest thanks to professor M. Nagumo for his kind advices and constant encouragement.

2. **Assumptions and Main Theorems. Assumptions.**

(I) The functions $A_{\mu\nu j}(t,x)$, $B_{\mu\nu}(t,x)$, $f_\mu(t,x)$ $(\mu,\nu=1,2,\cdots, k; j=1,2,\cdots,m)$ belong to $C_{(t,x)}$ on $D(T)$.

(II) The functions $A_{\mu\nu j}(t,x)$, $B_{\mu\nu}(t,x)$, $(\mu,\nu=1,\cdots,k; j=1,2,\cdots,m)$ belong to $A_{(x)}$ on $D(T)$ for each $t\in[0,T]$ and can be extended holomorphically with respect to x to the complex domain $\mathfrak{D}_\gamma(T)$ on which they are continuous, and on $\mathfrak{D}_\gamma(T)$, $|A_{\mu\nu j}(t,z)|\leqq A$, $|B_{\mu\nu}(t,z)|\leqq B$ where A and