123. Notes on Commutative Archimedean Semigroups. II

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This note is the continuation of [1] to report the results without proof. The same notations as those in [1] will be used without explanation.

6. Construction of the semigroups without idempotent.

Definition. An ordinary tree is a dispersed tree which satisfies the ascending chain condition and has at least one highest prime. An ordinary tree without smallest element is also called an ordinary tree of infinite length.

Theorem 8. *Assume that the following systems and functions are given:*

(15.1) *An abelian group G with a function I satisfying* (1.1) *through* (1.4).

(15.2) *A family $\{S_\lambda; \lambda \in G\}$ of ordinary trees of infinite length.*

(15.3) *A set $\{\iota_\lambda; \lambda \in G\}$ of highest primes.*

(15.4) *A commutative groupoid $(\cdot)$, $P = \bigcup_{\lambda \in G} P_\lambda$ with identity ι_ε where P_λ is the set of all primes of S_λ such that*

$$\text{for } \alpha_\lambda \in P_\lambda \text{ and } \beta_\mu \in P_\mu,\ \alpha_\lambda \cdot \beta_\mu \in P_{\lambda\mu}$$

and the following conditions are satisfied:

(16.1) $$\sigma(\alpha_\lambda)+\sigma(\beta_\mu)+I(\lambda, \mu)-\sigma(\alpha_\lambda\cdot\beta_\mu) \geqq h_{\alpha_\lambda\cdot\beta_\mu}(\alpha_\lambda\cdot\beta_\mu, \alpha_\lambda\cdot\beta'_\mu)-h_{\beta_\mu}(\beta_\mu, \beta'_\mu)$$
for all $\alpha_\lambda \in P_\lambda$, $\beta_\mu, \beta'_\mu \in P_\mu$, all $\lambda, \mu \in G$.

(16.2) $$\sigma(\alpha_\lambda)+\sigma(\beta_\mu)+\sigma(\gamma_\nu)+I(\lambda, \mu)+I(\lambda\mu, \nu) \geqq \sigma((\alpha_\lambda\cdot\beta_\mu)\cdot\gamma_\nu)+h_{(\alpha_\lambda\cdot\beta_\mu)\cdot\gamma_\nu}((\alpha_\lambda\cdot\beta_\mu)\cdot\gamma_\nu, \alpha_\lambda\cdot(\beta_\mu\cdot\gamma_\nu))$$
for all $\alpha_\lambda \in P_\lambda$, $\beta_\mu \in P_\mu$, $\gamma_\nu \in P_\nu$, $\lambda, \mu, \nu \in G$.

(16.3) *For any $\alpha_\lambda \in P_\lambda$ there is $m>0$ such that*
$$\sigma(\alpha_\lambda^{(m)})+\sigma(\alpha_\lambda)+I(\lambda^m, \lambda)-\sigma(\alpha_\lambda^m\cdot\alpha_\lambda)>0$$
where $\alpha_\lambda^{(m)}=\alpha_\lambda^{(m-1)}\cdot\alpha_\lambda$, $\alpha_\lambda^{(2)}=\alpha_\lambda\cdot\alpha_\lambda$

(16.1) *implies* (16.4) *below*:

(16.4) $\sigma(\alpha_\lambda)+\sigma(\beta_\mu)+I(\lambda, \mu)\geqq\sigma(\alpha_\lambda\cdot\beta_\mu)$.

Now we define a function $K(\alpha_\lambda, \beta_\mu)$ on $P\times P$ as follows:

(16.5) $K(\alpha_\lambda, \beta_\mu)=\sigma(\alpha_\lambda)+\sigma(\beta_\mu)+I(\lambda, \mu)-\sigma(\alpha_\lambda\cdot\beta_\mu)$.

Let $N\times P=\{(n, \alpha) : n\in N, \alpha\in P\}$ and let $S=(N\times P)/\xi$ where ξ is an equivalence defined by

$(n, \alpha)\xi(m, \beta)$ *if and only if α and β are in a same S_λ and*