122. Non-Connection Methods for Some Connection Geometries based on Canonical Equations of Hamiltonian Types of II-Geodesic Curves

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In [3], I established *non-connection methods* for linear connections *in the Large* bringing respective geometries to the "Erlanger Programm", the transformation group parameters being adequate functions of the (local) coordinates and in [4] he extended them further *doubly* to the case, where transformation group parameters are adequate functions of the (local) coordinates (x) as well as of $(\dot{x}, \ddot{x}, \cdots, \overset{(M)}{x})$, $(\dot{x}=dx/dt$, etc.; $t=$curve parameter). In [5], [6], and [8], M. Kurita studied the Finsler spaces by means of the canonical equations of Hamiltonian types. In this note, I will, being suggested by his means, establish the following geometries based on canonical equations of Hamiltonian types of the II-*geodesic curves* in my sense: (I) (Doubly) extended affine geometry, (II) (Doubly) extended Euclidean geometry, (III) Other 20 (doubly) extended geometries indicated on p. 247 of [14], (IV) Geometry of Finsler-Craig-Synge-Kawaguchi spaces, all based on canonical equations of Hamiltonian types of II-geodesic curves in the present author's sense. (IV) is a detailed exposition of the n-dimensional case of Art. 4 of [1].

I. (Doubly) Extended affine geometry based on canonical equations of Hamiltonian types of II-geodesic curves. **I.1.** *A new method of treatment of* II-*geodesic curves based on canonical equations of Hamiltonian types.* Consider

(I.1) $$\omega\overset{\text{def}}{=}\omega_{\mu}(x, \dot{x}, \cdots, \overset{(M)}{x})dx^{\mu}, \quad (\lambda, \mu, \cdots=1, 2, \cdots, n),$$

which is *global* in the differentiable manifold $M=\bigcup_{\alpha} U_{\alpha}$ of class $C^{\nu}(\nu=$positive integer or ∞ or $\omega)$, where the open subset U_{α} is the domain of the local coordinates (x), since (I.1) is written in an invariant form.

Let $x^{\lambda}=x^{\lambda}(t)$ be a parametrized curve, where t is the canonical parameter ([14], Art. 12; [15], Art. 14). Set

(I.2) $$d\xi\overset{\text{def}}{=}\omega_{\mu}(x, \dot{x}, \cdots, \overset{(M)}{x})\dot{x}^{\mu}dt,$$

(I.3) $$L=\omega_{\mu}(x, \dot{x}, \cdots, \overset{(M)}{x})\dot{x}^{\mu}=p_{\mu}\dot{q}^{\mu}, \quad (q^{\mu}=x^{\mu}).$$

Then the Lagrangian equations for the extremal problem