121. *On the Separation Theorem of Intermediate Propositional Calculi*

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In this paper is shown a sufficient condition for intermediate propositional calculi to have the separation theorem on them. By an intermediate propositional calculus we mean a calculus between the classical and the intuitionistic obtained by adding some new axioms to an intuitionistic propositional calculus. And by the separation theorem we mean the following

Theorem. *A provable formula in the calculus can be proved using only the axioms for implication and those for the logical symbols actually appearing in the formula.*

This theorem depends upon the axiom system of the calculus. And we call the calculus and its axiom system as separable if the separation theorem holds on the calculus. An example of separable intuitionistic systems is given in [3] and those of separable classical systems are in [4] and [5].

A formula is called an I (or C, or D, or N) formula if it contains only implication (or conjunction, or disjunction, or negation) as its logical symbols. An IC formula is a formula in which no logical symbols other than implication and conjunction are contained. An IC axiom is an axiom which is an IC formula. An IC theorem is a theorem which is an IC formula and is provable from IC axioms. An IC proof is a proof in which only IC axioms are used. A calculus is IC complete if the theorems which are IC formulas are IC theorems. And other combinations are defined similarly.

What is proved in this paper is that *if an intermediate propositional calculus satisfies the following* (1), (2), *and* (3), *it is separable.*

(1) *The axiom system of the calculus is constructed by adding some new* I *axioms to the axiom system of a separable intuitionistic propositional calculus. And the rule of substitution is in it.*

(2) *The calculus is* I *complete.*

(3) *There exist* I *formulas* $f_i(a, b)$ $(i=1, \cdots, n)$ *whose propositional variables are only* a *and* b *such that formulas of the forms*

$$a \vee b \supset f_i(a, b) \qquad (i=1, \cdots, n)$$