165. Some Applications of the Functional-Representations of Normal Operators in Hilbert Spaces. XXII

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Throughout the present paper we again treat the function $U(\lambda)\in\mathfrak{F}^*$ stated in Definition B and Theorem 60 of the preceding one; that is, $U(\lambda)$ is a function belonging to $\mathfrak{F}^*$ such that any point λ_ν of the denumerably infinite bounded set $\{\lambda_\nu\}_{\nu=1,2,3,\dots}$ assigned arbitrarily is an essential singularity of $U(\lambda)$ in the sense of the functional analysis, that the mutually disjoint closed domains $D_j(j=1, 2, 3, \cdots, n)$ with $\overline{\{\lambda_\nu\}}\cap\left[\bigcup_{j=1}^{n} D_j\right]=\phi$, assigned arbitrarily, form the sets of singularities of $U(\lambda)$ in the sense stated in Definition B and lie on the disc $|\lambda|\leqq\sup_\nu|\lambda_\nu|$, and that $U(\lambda)$ is regular in the complex λ-plane $\{\lambda: |\lambda|<+\infty\}$ except for $\overline{\{\lambda_\nu\}}\cup\left[\bigcup_{j=2}^{n} D_j\right]$.

Theorem 62. Let $\{\lambda_\nu\}$ be everywhere dense on a closed or an open rectifiable Jordan curve Γ; let the ordinary part of the function $U(\lambda)\in\mathfrak{F}^*$ stated above be a non-zero constant ξ; let c be an arbitrary complex number, finite or infinite; let $\sigma=\sup_\nu|\lambda_\nu|$; let $n(\rho, c)$ be the number (counted according to the respective multiplicities) of c-points of $U(\lambda)$ in the closed domain $\overline{\Delta}_\rho\{\lambda: \rho\leqq|\lambda|\leqq+\infty\}$ with $\sigma<\rho<+\infty$; let $\bar{n}(\rho, c)$ be the number of distinct c-points of $U(\lambda)$ in $\overline{\Delta}_\rho$; let

$$m(\rho, c)=\begin{cases}\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}\overset{+}{\log}\frac{1}{|U(\rho e^{-it})-c|}dt & (c\neq\infty)\\ \dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}\overset{+}{\log}|U(\rho e^{-it})|\,dt & (c=\infty),\end{cases}$$

$$N(\rho, c)=\int_\rho^{+\infty}\frac{n(r, c)-n(\infty, c)}{r}dr-n(\infty, c)\log\rho,$$

and

$$\overline{N}(\rho, c)=\int_\rho^{+\infty}\frac{\bar{n}(r, c)-\bar{n}(\infty, c)}{r}dr-\bar{n}(\infty, c)\log\rho$$

for any ρ with $\sigma<\rho<+\infty$; and let

$$\delta(c)=1-\overline{\lim_{\rho\to\sigma+0}}\frac{N(\rho, c)}{m(\rho, \infty)},$$

$$\Theta(c)=1-\overline{\lim_{\rho\to\sigma+0}}\frac{\overline{N}(\rho, c)}{m(\rho, \infty)},$$