164. *An Integral of the Denjoy Type.* II

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1. Introduction. The author [3] introduced the approximately continuous Denjoy integral (AD-integral) which is based on the descriptive definition of the general Denjoy integral.

The AD-integral is an extension of Burkill's approximately continuous Perron integral (AP-integral) [3] and of Denjoy's general integral (D-integral). In section 2 we shall state some fundamental properties of the AD-integral and it will be proved that our integral and the GM-integral defined by H. W. Ellis [1] are not compatible. An integral of the Perron type equivalent to the AD-integral is given in section 3.

2. The approximately continuous Denjoy integral. A real valued function $f(x)$ is said to be $\underline{AC}$ on a linear set E if, to each positive number ε, there exists a number $\delta>0$ such that

$$\sum\{f(b_k)-f(a_k)\}>-\varepsilon$$

for all finite non-overlapping sequences of intervals $\{(a_k, b_k)\}$ with end points on E and such that $\sum(b_k-a_k)<\delta$. There is a corresponding definition $\overline{AC}$ on E. If the set E is the sum of a countable number of sets E_k on each of which $f(x)$ is $\underline{AC}$ then $f(x)$ is said to be $\underline{ACG}$ on E. If the set E_k are assumed to be closed, then $f(x)$ is said to be $(\underline{ACG})$ on E. Similarly we can define $\overline{ACG}$ and $(\overline{ACG})$ on E. A function is (ACG) on E if it is both $(\underline{ACG})$ and $(\overline{ACG})$ on E.

Let $f(x)$ be a function defined on $[a, b]$ and suppose there exists a function $F(x)$ such that

(i) $F(x)$ is approximately continuous on [a, b],

(ii) $F(x)$ is (ACG) on [a, b],

(iii) $AD\ F(x)=f(x)$ a.e.,

then $f(x)$ is said to be integrable on $[a, b]$ in the approximately continuous Denjoy sense or AD-integrable. We than say that the function $F(x)$ is an indefinite AD-integral of $f(x)$ which is uniquely determined except an additive constant (Lemma 3.1 below).

The author [3] proved that the AD-integral is more general than Burkill's approximately continuous Perron integral (AP-integral) [6].

The condition in (ACG) that the set E_k be closed gives no restriction when $F(x)$ is continuous since the continuity of $F(x)$ is