163. Boundary Value Problems for the Helmholtz Equations. II

The Case of Parallel Lines with Openings

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1. In the preceding paper [1], the author has solved two kinds of boundary value problems for the Helmholtz equations in domains bounded by coaxial circles with arbitrary number of arbitrary openings in them. These correspond to the E and H electromagnetic fields in media, which are not necessarily the same, occupying contiguous coaxial circular domains separated by circular boundaries of perfect conductivity which having arbitrary slots in them. In this paper, the same approach is applied to solve the Helmholtz equations in domains separated by parallel lines with arbitrary openings in them. This corresponds to the E and H waves in media, which are not necessarily identical, in domains separated by parallel gratings of plane strips of arbitrary width.

The simplest problem of diffraction of electromagnetic waves by a grating is that when the grating is composed of (i) a single, (ii) infinitely long series of (iii) equally spaced obstacles of a regular geometry, (iv) in a uniform medium and when (v) a plane wave is incident on it. If we replace one or more of these conditions (i)–(v) by some other conditions, then the problem will be generalized in various ways. For example, (ii) has been generalized to a case of a grating of finite or semi-infinite series of an identical cylinder [2].

In this paper, the conditions (i), (iii), and (iv) are generalized and it is assumed that there are two parallel gratings of plane strips, composed of arbitrary number of line segments of arbitrary length, and that the media separated by these gratings are not necessarily identical. Note that the method is generalized, in a way similar to that mentioned in the previous paper [1], to the case where the number of gratings is more than two. In this paper, it is also assumed that the gratings are of periodic structure in the large, though they may be non-periodic locally. The result is expected to be generalized to the case where there is no such periodicity.

2. Let b and d be positive real numbers, and that L_j $(j=1, 2)$