162. *Boundary Value Problems for the Helmholtz Equations. I*

The Case of Coaxial Circular Arcs

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1. Let (r, θ) be polar coordinates in a plane and let S_j be domains defined by $S_1; r<a_1$, $S_j; a_{j-1}<r<a_j$, $(j=2, 3, \cdots, \nu)$, $S_{\nu+1}$; $a_\nu<r$. $(a_1<a_2<\cdots<a_\nu)$. Suppose that, for each $j=1, 2, \cdots, \nu$, L_j is a union of arbitrary (but finite) number of circular arcs of arbitrary width and of radius a_j, and that L_j^c is the complement of L_j with respect to the whole circle $r=a_j$. Then, our problems are stated as follows; Find functions $u_j(r, \theta)$ in S_j such that their partial derivatives of the second order are continuous in S_j excepting given points $x_j^* \in S_j$, that u_j and $\partial u_j/\partial r$ are Hölder continuous in the closure of S_j, and that they satisfy

(1) $\Delta u_j+k_j^2 u_j=f_j\delta(x, x_j^*), \quad x \in S_j,\ x_j^* \in S_j, \qquad (j=1, 2, \cdots, \nu+1)$

(2) u and $\dfrac{\eta}{k}\dfrac{\partial u}{\partial r}$ are continuous when they traverse L_j^c.

(3) $$\lim\cdot r^{\frac{1}{2}}\left\{\frac{\partial u_{\nu+1}}{\partial r}+ik_{\nu+1}u_{\nu+1}\right\}=0, \qquad r\to\infty,$$

and

(4) $u_j=0$ on $L_{j-1}+L_j$, $(j=2,3,\cdots,\nu)$, $u_1=0$ on L_1 and $u_{\nu+1}=0$ on L_ν,

or

(4)′ $$\frac{\partial u_j}{\partial r}=0 \text{ on } L_{j-1}+L_j, \qquad (j=2, 3, \cdots, \nu),$$

$$\frac{\partial u_1}{\partial r}=0 \text{ on } L_1 \text{ and } \frac{\partial u_{\nu+1}}{\partial r}=0 \text{ on } L_\nu,$$

where Δ is the two-dimensional Laplace operator and k_j $(j=1, 2, \cdots, \nu+1)$ are complex constants where $\mathrm{Im}\cdot k_j \leqq 0$. $\eta=\eta_j$ and f_j are given complex constants where f_j may include zero but $\eta_j \neq 0$. These are the simultaneous boundary value problems for the Helmholtz equations in contiguous domains bounded by circular arcs, in which the parameters k_j are not necessarily uniform. They are a generalization of the boundary value problem for the single Helmholtz equation, and is also a generalization of the theory of electromagnetic fields in a uniform medium bounded by circular arcs