160. On P^* Spaces and Equicontinuity

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Let P be a topological property.[1] A topological space X is called a *P^* space* if a subset U of X is open in X whenever $U \cap K$ is open in K for any subset K in X satisfying P. The purpose of this note is to investigate properties of P^* spaces and as applications to obtain some extensions of a theorem of Gleason [2] and the Ascoli's theorem.

1. Let E be a set, then we can consider the lattice of all topologies on E, that is, the ordering of the lattice can be defined as follows; $X \geqq Y$ if $O(X) \supset O(Y)$, where $O(X)$ (or $O(Y)$) is the set of all open subsets in X (or Y). For any family $\{X_j\}$ of topological spaces on E, $\vee X_j$ or $\wedge X_j$ denotes the join or the meet of $\{X_j\}$ ([4], [6]). A topological property P is said to have the *condition* (C) if it satisfies the following condition; any space consisting of one point has P, and any continuous image of X also satisfies P if a topological space X has P. Examples of topological properties having (C) are "compact", "separable", "connected", and "arcwise connected",[2] and any k-space ([5]) is a P^* space, where P is "compact".

We first prove the following theorem.

1.1. **Theorem.** *Let a topological property P have (C). If $\{X_\alpha\}$ are P^* spaces on the same basic set, then $\wedge X_\alpha$ is also a P^* space.*

Proof. Put $Z = \wedge X_\alpha$, then Z is a quotient space (cf. [5]) of $\sum X_\alpha$, where $\sum X_\alpha$ denotes the sum of X_α.[3] Since $\{X_\alpha\}$ are P^* spaces, it is clear that $\sum X_\alpha$ is a P^* space, so by the next lemma, the theorem is proved.

1.2. **Lemma.** *Let P be a topological property satisfying (C). If X is a P^* space then any quotient space of X is also a P^* space.*

Proof. The lemma can be proved easily.

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1) Let P be a property of topological spaces. P is said to be topological if it is invariant under homeomorphisms.

2) X is arcwise connected if for two points a, b in X there is a continuous image of closed interval containing a, b in X.

3) The fact is due to Professor K. Morita. In $\sum X_\alpha$, $\{X_\alpha\}$ are mutually disjoint and any X_α is open in $\sum X_\alpha$.