158. *On the Existence of Discontinuous Solutions of the Cauchy Problem for Quasi-Linear First-Order Equations*

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1. **Introduction.** In recent years, interest in discontinuous solutions of the Cauchy problem for nonlinear partial differential equations has conciderably increased and much progress has been made for quasi-linear first-order equations of conservation type in a single space variable (see Oleinik [3] for a survey of literatures).

In the case of several space variables, using a finite difference scheme, Conway and Smoller [1] has proved the existence of weak solutions of the Cauchy problem

$$u_t+\sum_{i=1}^{n}\frac{\partial f^i(u)}{\partial x_i}=0 \tag{1.1}$$

with a bounded measurable initial function having locally bounded variation in the sense of Tonelli-Cesari. A function f is said to have locally bounded variation in the sense of Tonelli-Cesari over R^n if for any compact set K in R^n there exists a set N of measure zero such that

$$V^i(x_1,\cdots,x_{i-1},x_{i+1},\cdots,x_n)=\underset{K-N}{\operatorname{Var}} f(x_1,\cdots,x_{i-1},\cdot,x_{i+1},\cdots,x_n),\ i=1,\cdots,n$$

is measurable and summable, and we denote by F the class of these functions.

The purpose of this paper is to prove the existence of weak solutions of the Cauchy problem of the type:

$$u_t+\sum_{i=1}^{n}\frac{\partial}{\partial x_i}f^i(t,x,u)+g(t,x,u)=0, \tag{1.2}$$

$$u(0,x)=u_0(x)\in F. \tag{1.3}$$

For simplicity, we restrict ourselves to the case $n=2$. But it will be easily seen that one can extend at once everything which we do in this case to the case $n\geqq 3$. Thus we shall consider the Cauchy problem

$$u_t=\frac{\partial}{\partial x}f(t,x,y,u)+\frac{\partial}{\partial y}g(t,x,y,u)+h(t,x,y,u)=0, \tag{1.4}$$

$$u(0,x,y)=u_0(x,y)\in F, \tag{1.5}$$

in the region

$$G=\{(t,x,y);\ 0\leqq t\leqq T<\infty,\ -\infty<x,y<\infty\}.$$

We call a function $u(t,x,y)$ a weak solution of (1.4), (1.5) if it