157. *On J-Groups of Spaces which are Like Projective Planes*

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(Comm. by Zyoiti SUETUNA, M.J.A., Sept. 12, 1966)

Let K be a CW-complex obtained from attaching a $2n$-cell V^{2n} to the n-sphere S^n by a map $f\colon S^{2n-1}\to S^n$. We call K a space which is like real, complex, quaternian, Cayley projective plane in accordance with $n=1, 2, 4, 8$. Our purpose is to calculate J-groups of $K^{(*)}$. Since J-group of a space is determined by its homotopy type we shall use the following notations:

$$P_R(m)=S^1\underset{f}{\smile}e^2,\quad (f)\in\pi_1(S^1)=Z[\iota],\quad (f)=m[\iota]$$
$$P_C(m)=S^2\underset{f}{\smile}e^4,\quad (f)\in\pi_3(S^2)=Z[h],\quad (f)=m[h]$$
$$P_Q(m, n)=S^4\underset{f}{\smile}e^8,\quad (f)\in\pi_7(S^4)=Z[\nu]+Z_{12}[\tau],\quad (f)=m[\nu]+n[\tau]$$
$$P_K(m, n)=S^8\underset{f}{\smile}e^{16},\ (f)\in\pi_{15}(S^8)=Z[\sigma]+Z_{120}[\rho],\ (f)=m[\sigma]+n[\rho]$$

where $[\iota]$, $[h]$, $[\nu]$, $[\tau]$, $[\sigma]$, $[\rho]$ are the generators of respective homotopy groups and $[\iota_4, \iota_4]=2[h]+[\tau]$, $[\iota_8, \iota_8]=2[\sigma]+\rho$.

For example $P_R(2)$, $P_C(1)$, $P_Q(1, 0)$, $P_K(1, 0)$ have respectively the same homotopy type as real, complex, quaternion, Cayley projective planes. Now let $\widetilde{KO}(X)$ denote the abelian group formed by all stable real vecter bundles over X. Then there exists the natural onto-homomorphism $J\colon \widetilde{KO}(X)\to J(X)$ by the definition of $J(X)$. Hence in order to determine $J(X)$ it is sufficient to calculate $\widetilde{KO}(X)$ and the kernel of J.

1. Case of $P_R(m)$. If m is odd we have $\widetilde{KO}(P_R(m))$ is trivial and therefore $J(P_R(m))$ is also trivial. If m is even we have $J^{-1}(0)=0$ by the following

Lemma 1. *The commutative diagram is exact:*

$$\begin{array}{ccccccccc}
 & & 0 & & & & 0 & & \\
 & & \downarrow & & & & \downarrow & & \\
0 & \longrightarrow & \widetilde{KO}(S^2) & \underset{p^*}{\longrightarrow} & \widetilde{KO}(P_R(m)) & \underset{i^*}{\longrightarrow} & \widetilde{KO}(S^1) & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & J(S^2) & \underset{p^*}{\longrightarrow} & J(P_R(m)) & \underset{i^*}{\longrightarrow} & J(S^1) & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array}$$

(*) J. F. Adames: On the group $J(X)$-1, Topology, Vol. 2 (1963).