200. *Classification and the Characters of Irreducible Representations of $SU(p, 1)$*

By Takeshi HIRAI

Department of Mathematics, Kyoto University

(Comm. by Kinjirô KUNUGI, M.J.A., Oct. 12, 1966)

Let $SU(p, 1)$ be the matrix group defined by

$$g \in GL(p+1, c),\ \det g = 1,\ {}^t\bar{g}Jg = J, \tag{1}$$

where $J = \begin{bmatrix} 1_p & \\ & 1 \end{bmatrix}$ and 1_p is the $p \times p$ unit matrix.

In this note, we intend to classify all completely[1] (or more generally, quasi-simple[2]) irreducible representations of this group and to obtain explicite formulae of their characters.

§1. Sketch of classification of irreducible representations. It is convenient to handle, in place of $SU(p, 1)$, the group G which is defined by ${}^t\bar{g}Jg = J$ ($\det g = 1$ is not assumed). The results on $SU(p, 1)$ are easily deduced from the results on G.

A maximal compact subgroup U of G is the totality of matrices of the form $\begin{bmatrix} u & \\ & \lambda \end{bmatrix}$, $u \in U(p)$ and $|\lambda| = 1$. Every completely (or quasi-simple) irreducible representation, if it is restricted to U, contains any irreducible representation of U at most once [2], [6], [4, a, II].

To every quasi-simple irreducible representation, there exists a completely irreducible representation which is infinitesimally equivalent[2] to it. In the following, we say irreducible in place of quasi-simple irreducible for simplicity. There corresponds to every infinitesimally equivalent class of irreducible representations of G an algebraically equivalent class of algebraically irreducible representations of the Lie algebra $\mathfrak{G}$ of G [2], [4, a, I]. Therefore the classification of all infinitesimally equivalent classes of irreducible representations of G is reduced to the classification of algebraically irreducible representations of $\mathfrak{G}$ which is $\mathfrak{U}$-simple ($\mathfrak{U}$ is the Lie algebra of U), that is, any (finite dimensional) irreducible representation of $\mathfrak{U}$ is contained in it at most once.

Then we are lead to commutation relations between some difference operators in a vector space. The situation is quite similar to the case of the Lorentz group of n-th order [5, a, b], [6]. Using the results in [1], we obtain a system of difference equations. We can obtain all solutions of the difference equations, which satisfy some conditions expressing irreducibility of representations of $\mathfrak{G}$.

---

1) See [2]. 2) See [4, a, I].