196. A Probabilistic Treatment of Semi-Linear Parabolic Equations

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Recently N. Ikeda, M. Nagasawa, and S. Watanabe have given a definition of branching Markov processes in general set up and have gotten several results about their structure [1], [2], and [3]. The purpose of this paper is to extend their methods in order to give a probabilistic treatment for semi-linear parabolic equation $\frac{\partial u}{\partial t}=\frac{1}{2}\Delta u+F(u)$ which was discussed by A. Kolmogoroff, I. Petrovsky, and N. Piscounoff [5] (abbreviated as KPP-equation). When we deal with KPP-equation, one of the difficulties comes from the fact that some coefficients of $F(u)$ may be negative even when $F(u)$ is a polynomial. In general, it happens that the solution of a semi-linear parabolic equation takes negative values and infinite values even for a positive bounded initial value. So if we want to treat it in probabilistic way, we must introduce some artificial procedure. One possible way is perhaps to permit the fundamental probability measure of the process to take signed values and infinite total mass. But we do not take this way. In this paper it is solved by introducing two kinds of technical operation, but it will be seen that they have natural intuitive probabilistic interpretation. One of them is to extend the state space of the processes in appropriate way, and the other is to make an operation to the initial value (cf (1.1) and (3.2)).

1. Notations and definitions. Following [2], we introduce some notations. Let R_d be d-dimensional Euclidian space, $\bar{R}_d$ be the one-point compactification of R_d, and let $N=\{0, 1, 2, 3, \cdots\}$. Also, let $S=\bar{R}_d\times N$ be the topological sum of $\bar{R}_d\times\{i\}$, $i\in N$. We denote the n-fold product of S with itself by $S^{(n)}$ and we say that $\boldsymbol{z}=((x_1, k_1), (x_2, k_2), \cdots, (x_n, k_n))\in S^{(n)}$ is R-equivalent to $\boldsymbol{z}'=((x'_1, k'_1), (x'_2, k'_2), \cdots, (x'_n, k'_n))\in S^{(n)}$, if $(x_1, x_2, \cdots, x_n)$ is obtainable by a permutation of $(x'_1, x'_2, \cdots, x'_n)$ and if $k_1+k_2+\cdots+k_n=k'_1+k'_2\cdots+k'_n$. Let us denote the quotient spaces $S^{(n)}/R$ by S^n, and write $\boldsymbol{z}=(\boldsymbol{x}, \boldsymbol{k})$ if $\boldsymbol{x}=(x_1, x_2, \cdots, x_n)$ and $\boldsymbol{k}=(k_1, k_2, \cdots, k_n)$. In the following, we write as $|\boldsymbol{k}|=k_1+k_2+\cdots+k_n$.

Now, $\bar{R}^n_d=\{\boldsymbol{x}; \boldsymbol{z}=(\boldsymbol{x}, \boldsymbol{k})\in S^n\}$ and S^n are metric spaces. Let us