

193. On Axiom Systems of Propositional Calculi. XXIII

By Kiyoshi ISÉKI

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In our papers ([1], [5]), by using J. Lukasiewicz method, we proved that the Russell system:

- 1 $CpCqp$,
- 2 $CCpqCCqrCpr$,
- 3 $CCpCqrCqCpr$,
- 4 $CNNpp$,
- 5 $CCpNpNp$,
- 6 $CCpNqCqNp$

is equivalent to the classical propositional calculus.

In my paper [2], the propositional calculus satisfying the conditions 1-3, 5 and 6 mentioned above is called a *NB-system*. For any implicational calculus not containing the negation functor N , we introduce the symbol '0' as a propositional constant, and define Np as $Cp0$ (for details, see [4], pp. 50-51).

As well known, an axiom system of the positive implicational calculus is given by J. Lukasiewicz as follows:

- 7 $CpCqp$,
- 8 $CCpCqrCCpqCpr$.

In our paper [1], we deduced some theses from 7 and 8. For example, we proved the following theses:

- 9 $CCpCqrCqCpr$,
- 10 $CCpqCCqrCpr$,
- 11 $CCpCpqCpq$.

We define

- 12 $Np = Cp0$,

where 0 is a propositional constant.

$$9 \text{ } r/0 \text{ } *C12-13,$$

- 13 $CCpNqCqNp$.

$$11 \text{ } q/0 \text{ } *C12-14,$$

- 14 $CCpNpNp$.

Therefore we have the *NB-system*.

If we add two axioms:

- 15 $CCqpCCCpqqp$

and Wajsberg axiom

- 16 $C0p$,

then as already shown in A. N. Prior ([4], p. 51), by these axioms we have