188. *A Series of Successive Modifications of Peirce's Rule*

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After Ono [2], we denote by LOS the sentence-logical part of the *primitive logic* LO [1]. LOS is the logic having $\rightarrow$ (*implication*) as the only logical constant. We may axiomatize LOS as follows:

(1) $p\rightarrow(q\rightarrow p)$,

(2) $(p\rightarrow(q\rightarrow r))\rightarrow((p\rightarrow q)\rightarrow(p\rightarrow r))$,

with substitution and detachment (*modus ponens*) as the only rules of inference. (p, q, r are three distinct proposition-variables.) Next, we denote by LOQS a logic obtained from LOS by adding

(3) $((p\rightarrow q)\rightarrow p)\rightarrow p$, (*Peirce's rule* [3]),

to the axioms of LOS. We can easily see that Peirce's rule is not provable in LOS. Hence, LOS is weaker than LOQS. (Notation: LOS$\subset$LOQS.)

On the advice of Prof. K. Ono, we studied the following problem: "Does there exist a logic L such that LOS$\subset$L$\subset$LOQS?" This problem has been solved in the affirmative. Namely, we have recognized the fact that we can obtain a series of successive modifications of Peirce's rule, by substituting the foregoing modified Peirce's rule in place of q in the proposition (3) (Peirce's rule) over and over again renewing p each time. The purpose of the present paper is to introduce a method for weakening Peirce's rule and to give a series of successive modifications of Peirce's rule. The author would wish to express his thanks to Prof. K. Ono for his kind guidance and encouragement.

§1. To begin with, we explain a first step of the above-mentioned method. In order to prove that the proposition (3) is not provable in LOS, we usually make use of the matrix[1] $N=\langle\{0, 1, 2\}, \{0\}, \rightarrow_N\rangle$, where

$$a\rightarrow_N b=\begin{cases} b \text{ if } a<b, \\ 0 \text{ otherwise.}\end{cases}$$

Namely, the propositions (1) and (2) are satisfied by N, but (3) is not satisfied by N. (Here, a proposition P is said to be satisfied by N if and only if P takes the value 0 identically with respect to N.) In fact, we can easily see the following:

1) As for matrices, see Rose [4] for example.