227. On an Addition Theorem for M-Spaces

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1. Introduction. Let X be a topological space and $\{A_\alpha\}$ a locally finite closed covering of X. As is well known, if each subspace A_α has one of the following properties, then the whole space X has also the same property (see, for instance, K. Morita [4] and J. Nagata [8]):

(a) being a normal space, (b) being countably paracompact,
(c) being collectionwise normal, (d) being perfectly normal,
(e) being paracompact and normal, (f) being metrizable.

Recently, K. Morita [7] has introduced the notion of M-spaces. He calls a topological space X an *M-space* if there exists a normal sequence $\{\mathfrak{U}_n \mid n=1, 2, \cdots\}$ of open coverings of X satisfying the condition (*) below:

(*) If a family $\mathfrak{K}$ consisting of a countable number of subsets of X has the finite intersection property and containing as its member a subset of $\mathrm{St}(x_0, \mathfrak{U}_n)$ for each n and for some fixed point x_0 in X, then $\bigcap\{\bar{K} \mid K\in\mathfrak{K}\}\neq\emptyset$.

In this note, we shall establish an analogous result for the notion of M-spaces; namely, we shall prove the following theorem:

Theorem 1. *Let $\{A_\alpha\}$ be a locally finite covering of a Hausdorff space X and each A_α be a closed G_δ-subset of X. If each A_α is a normal M-space with respect to its relative topology, then the whole space X is also a normal M-space.*

The next § 2 is devoted to the proof of this theorem, and in § 3 we shall deduce some of its immediate consequences. Most terminologies and notations used in this note are the same as those of J. W. Tukey [12].

Finally, I wish to express my hearty thanks to Prof. K. Morita who has given me many kindful suggestions and advices.

2. Proof of Theorem 1. Our proof of Theorem 1 rests upon the following two lemmas.

Lemma 1. *Let $\{\mathfrak{G}_n \mid n=1, 2, \cdots\}$ be a normal sequence of open coverings of a topological space X. Then there is another normal sequence $\{\mathfrak{H}_n \mid n=1, 2, \cdots\}$ of open coverings of X having the properties:*

(i) *Each $\mathfrak{H}_n$ is a refinement of $\mathfrak{G}_n$.*