# 224. Integration on Locally Compact Spaces Generated by Positive Linear Functionals Defined on the Space of Continuous Functions with Compact Support and the Riesz Representation Theorem.*) I

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A non-empty family $V$ of sets of an abstract space $X$ is called a *prering* if the following condition is satisfied: if $A, B \in V$ then $A \cap B \in V$ and there exists disjoint sets $C_1, \cdots, C_k \in V$ such that $A \backslash B = C_1 \cup \cdots \cup C_k$.

A function $\mu$ from a prering $V$ into a Banach space $Z$ is called a vector volume if it satisfies the following condition: for every countable family of disjoint sets $A_t \in V (t \in T)$ such that

(a) $$A = \bigcup_T A_t \in V$$

we have $\mu(A) = \sum_T \mu(A_t)$, where the last sum is convergent absolutely and the variation of the function $\mu$, that is, the function

$$|\mu|(A) = sup \left\{\sum_T |\mu(A_t)|\right\}$$

is finite for every set $A \in V$, where the supremum is taken over all possible decompositions of the set $A$ into the form (a). A volume is called positive if it takes on only non-negative values. If $\mu$ is a volume then its variation $|\mu|$ is a positive volume.

If $v$ is a volume on a prering $V$ of subsets of a space $X$ then the triple $(X, V, v)$ is called a volume space.

Let $R$ be the space of reals and $Y, Z, W$ be Banach spaces. Denote by $X$ the space of all bilinear continuous operators $u$ from the space $Y \times Z$ into the space $W$. Norms of elements in the spaces $Y, Y', Z, W, U$ will be denoted by $|\ \ |$.

In the paper [1] has been presented an approach to the theory of the space $L(v, Y)$ of Lebesgue-Bochner summable functions generated by a positive volume $v$. The construction was not based on measure or on measurable functions. It allowed us to prove the basic structure theorems of the space of summable functions and at the same time to develop the theory of an integral of the form $\int u(f, d\mu)$, where $u$ denotes a bilinear continuous operator from $U = L(Y, Z; W)$, $f \in L(v, Y)$, and $\mu$ is a finitely additive func-

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