223. *Some Notes on the Cluster Sets of Meromorphic Functions*

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1. Let D be a domain in the z-plane, Γ its boundary, E a totally disconnected compact set on Γ and z_0 a point of E such that $U(z_0)\cap(\Gamma-E)\neq\emptyset$ for any neighborhood $U(z_0)$ of z_0. We consider a normal exhaustion $\{F_n\}$ of the complementary domain F of E with respect to the extended z-plane and the graph $0<u<R$, $0<v<2\pi$ associated with this exhaustion in Noshiro's sense [3], where R is the length of this graph and may be infinite. The niveau curve $u(z)=r(0<r<R)$ on F consists of a finite number of closed analytic curves $\gamma_r^{(i)}(i=1, 2, \cdots, m(r))$ and we set

$$\Lambda(r)=\max_{1\leq i\leq m(r)}\int_{\gamma_r^{(i)}}dv.$$

Now suppose that there exists an exhaustion $\{F_n\}$ with the graph satisfying

$$(1)\qquad \limsup_{r\to R}\,(R-r)\int_0^r\frac{dr}{\Lambda(r)}=\infty.$$

Then the integral $\int_0^R\exp\left(2\pi\int_0^r\frac{dr}{\Lambda(r)}\right)dr$ diverges, so that the complementary domain F of E belongs to the class O_{AB}^0 (see Kuroda [1]), i.e., E belongs to the class $N_{\mathfrak{B}}^0$ in the sense of Noshiro [4]. Therefore, for any single-valued meromorphic function $w=f(z)$ in D, the set $\Omega=C_D(f, z_0)-C_{\Gamma-E}(f, z_0)$ is empty or open and each value α belonging to $\Omega-R_D(f, z_0)$ is an asymptotic value of $w=f(z)$ at z_0 or there is a sequence of points $\zeta_n\in E$ tending to z_0 such that α is an asymptotic value of $f(z)$ at each ζ_n. Further $\Omega-R_D(f, z_0)$ is an at most countable union of sets of the class $N_{\mathfrak{B}}$. (These three facts have been proved by Noshiro in his recent paper [4].) We shall restrict our consideration to the case where E is contained in a single boundary component Γ_0 of Γ. Then we have

Theorem 1. *Suppose that Ω is not empty. If E is contained in a single boundary component Γ_0 of Γ and there exists an exhaustion $\{F_n\}$ with the graph satisfying* (1), *then $w=f(z)$ takes on every value, with two possible exceptions, belonging to any component Ω_n of Ω, infinitely often in the intersection of any neighborhood of z_0 and D.*