222. On Branching Semi-Groups. II

By Nobuyuki IKEDA, Masao NAGASAWA,
and Shinzo WATANABE

Osaka University, Tokyo Institute of Technology, and Kyoto University*)

(Comm. by Kinjirô KUNUGI, M.J.A., Nov. 12, 1966)

We have discussed in [4] and [5] on the methods of the construction of a branching Markov process. The purpose of this paper is to give another analytic method of construction based on S-equation.[1] To do this we shall first construct a solution of S-equation with a initial value by the usual method of successive approximation and then we shall define a branching semi-group with the aid of these solutions. It will turn out that this semi-group coincides with that constructed in [5] by the method of Moyal [7]. This fact follows from a result of [5] that the semi-group constructed in [5] by the method of Moyal is a branching semi-group. (the proof depends essentially on the Theorem 1 of [2].) But we shall give still another proof based on the uniquness of the solution of the forward equation.[2] This may be considered as a generalization of a method of Harris [6] to prove that (π_{ij}, q_j)-minimal Markov chain on $Z^+=\{0, 1, 2, 3, \cdots\}$ where $\pi_{ij}=p_{j-i+1}$ and $q_j=jb$[3] is a branching Markov process, i.e. its transition probability $\{p_{ij}(t)\}$ satisfies

$$\sum_{j=0}^{\infty} p_{ij}(t)s^j=\left[\sum_{j=0}^{\infty} p_{1j}(t)s^j\right]^i, \qquad \text{for every } 0<s\leqq 1.$$

Let S be a compact metrizable space and $\boldsymbol{S}=\bigcup_{n=0}^{\infty} S^n\cup\{\Delta\}$ be defined as in [2]. Let T_t be a positive strongly continuous semi-group on $\boldsymbol{C}(S)$ such that $T_t 1=1$ and take $k\in \boldsymbol{C}(S)^+$. Let $\mathfrak{G}$ be the infinitesimal generator of T_t in the Hille-Yosida sense and $\mathfrak{D}(\mathfrak{G})$ be the domain of $\mathfrak{G}$. Then it is well known that there exists uniquely a positive strongly continuous semi-group T_t^0 on $\boldsymbol{C}(S)$ such that $T_t^0 1\leqq 1$ and its generator $\mathfrak{G}^0$ is given by

(1) $$\mathfrak{G}^0=\mathfrak{G}-k \quad \text{and} \quad \mathfrak{D}(\mathfrak{G}^0)=\mathfrak{D}(\mathfrak{G}).$$ [4]

---

*) The authors' present addresses: Stanford Univ., Cornell Univ., and Univ. of Washington.

1) Cf. [3]. In the following we use the terminology and the notation of [2], [3], [4], and [5].

2) Cf. [3].

3) $\{p_i\}, i=0, 1, 2, 3, \cdots$ is a given probability sequence and b is a given positive constant.

4) A probabilistic method to obtain T_t^0 from the given T_t and k is the killing defined by the multiplicative functional $\exp\left(-\int_0^t k(x_s)ds\right)$.