# 253. *Boolean Multiplicative Closures*. I

By Roberto CIGNOLI
Instituto de Matemática, Universidad Nacional del Sur,
Bahia Blanca, Argentina

(Comm. by Kinjirô KUNUGI, M.J.A., Dec. 12, 1966)

0. **Introduction.** We shall say that a multiplicative closure operator $\nabla$ defined on a distributive lattice $L$ with zero and unit is a *Boolean multiplicative closure operator* if any closed element under $\nabla$ has a complement in $L$. Examples of Boolean multiplicative closure operators are the *possibility operator* defined by Gr. Moisil ([7], [8])[1] in (three-valued) Lukasiewicz algebras (see also [3] and [4]), and the operator $D_1$ defined by G. Epstein ([5], Definition 2) in Post algebras.

The aim of this note is to give a characterization of those distributive lattices (with zero and unit) that admits a Boolean multiplicative closure operator. In §1 we give the definitions and notations. In §2 we characterize additive-multiplicative closure operators by the set of their closed elements and in §3 we apply the results of §2 to solve our main problem. Finally, in §4 we show how some of the previous theorems can be extended to general multiplicative closure operators.

These results have some applications in the study of the lattice theory of many-valued logics. We were inspired in A. Monteiro's work on the ideal theory of (three-valued) Lukasiewicz algebras, that will be published elsewhere.

1. **Definitions and notations.** Let $L$ be a distributive lattice with zero 0 and unit 1. we shall consider operators $\nabla$ from $L$ into $L$ satisfying some of the following conditions:

C0) $\nabla 0=0$, C1) $x\leq \nabla x$, C2) $\nabla x=\nabla\nabla x$,
C3) If $x\leq y$, then $\nabla x\leq \nabla y$, C4) $\nabla(x\vee y)=\nabla x\vee \nabla y$,
C5) $\nabla(x\wedge y)=\nabla x\wedge \nabla y$.

If $\nabla$ satisfies C1), C2), and C3) it is called a *closure operator* (see [9], [12], [2]), and we shall denote the set of all closure operators on $L$ by $C(L)$.

If $\nabla$ satisfies C1), C2), and C4), (or C1), C2), and C5)), it is called an additive closure operator ([10], [11], [6]) (or multiplicative closure operator, [1], [6]), and we shall denote by $\mathrm{Ca}(L)(\mathrm{Cm}(L))$ the set of additive (multiplicative) closure operators defined on $L$. It is clear

1) The references are contained in the second paper.