252. *An Algebraic Formulation of K-N Propositional Calculus*

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A *K-N* axiom system of propositional calculus is given by J. B. Rosser (2). His axiom system of classical propositional calculus is written in the form of

a) *CpKpp*,

b) *CKpqp*,

c) *CCpqCNKqrNKrp*,

where functors *K*, *N*, *C* denote conjunction, negation, and implication respectively.

As well known, we have $Cpq=NKpNq$. Therefore Rosser's axiom system is denoted by two functors *K*, *N* as follows:

a′) *NKpNKpp*,

b′) *NKKpqNp*,

c′) *NKNKpNqNNKNKqrNNKrp*.

On the other hand, B. Sobociński obtained two new axiom systems which is equivalent to Rosser's system (see B. Sobociński [3], [4]). C. A. Meredith gave an axiom system (see C. A. Meredith and A. N. Prior [1]).

In the *K-N* propositional calculus, there are two rules of procedure:

1) One of them is the rule of substitution commonly used in the propositional calculus.

2) The other is the rule of detachment as follows. If $NK\alpha N\beta$ and α are theses, then β is also a thesis.

From Rosser's system or *KN*-system, we can define an algebraic system as follows: Let x be an abstract algebra consisting of $0, p, q, r, \cdots$ with a binary operation $*$ and a unary operation $\sim$ satisfying the following conditions:

1) $\sim(p*p)*p=0$,

2) $\sim p*(q*p)=0$,

3) $\sim\sim(\sim\sim(p*r)*\sim(r*q))*\sim(\sim q*p)=0$,

4) Let α, β be expressions in X, then $\sim\sim\beta*\sim\alpha=0$ and $\alpha=0$ imply $\beta=0$.

Then X is called *KN-algebra*. The condition 4) corresponds to the rule of detachment.

First of all, we shall prove some general theorems. The Greek