250. *On Certain Condition for the Principle of Limiting Amplitude*

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1. Introduction and results. We consider the nonstationary problems

$$\left[\frac{\partial^2}{\partial t^2} - \Delta + q(x)\right] u(x, t) = f(x) e^{-i\sqrt{\lambda}t} \qquad (\lambda > 0), \tag{1}$$

$$u(x, 0) = 0, \qquad \frac{\partial}{\partial t} u(x, 0) = 0; \tag{2$'$}$$

$$\left[\frac{\partial^2}{\partial t^2} - \Delta + q(x)\right] u(x, t) = 0, \tag{1$'$}$$

$$u(x, 0) = g_1(x), \qquad \frac{\partial}{\partial t} u(x, 0) = g_2(x); \tag{2}$$

in 3 Euclidean space R^3, where $q(x)$ is a real-valued function belonging to $C_0^2(R^3)$. Furthermore assume that the operator $L = -\Delta + q(x)$ has no eigenvalue. Here Δ denotes the Laplacian $\partial^2/\partial x_1^2 + \partial^2/\partial x_2^2 + \partial^2/\partial x_3^2$, and L is the unique self-adjoint extension in $L^2(R^3)$ of $-\Delta + q$ defined on $C_0^\infty(R^3)$. Then under the conditions imposed on q, L is strictly positive, and it is known that $D(L) = W_2^2(R^3)$, where $W_2^2(R^3)$ denotes the space of functions whose partial derivatives of order $\leqq 2$ in the sense of distribution belong to $L^2(R^3)$.

Then we have the following

Theorem 1. *Suppose that* $g_1(x) \in C_0^2(R^3)$, $g_2(x) \in C_0^1(R^3)$, *and* $f(x) \in C_0^1(R^3)$. *Then the following three conditions are equivalent:*

i) *The solution of the problem* (1), (2)$'$ *is such that at every point* $x \in R^3$ *we have*

$$\lim_{t \to \infty} u(x, t) e^{i\sqrt{\lambda}t} = u_+(x, \lambda) \qquad (\lambda > 0),$$

where $u_+(x, \lambda)$ *denotes* $\lim_{\varepsilon \to +0} u_\varepsilon(x, \lambda)$ *and* $u_\varepsilon(x, \lambda)$ *is the solution of the equation*

$$Lu = (\lambda + i\varepsilon)u + f.$$

ii) *The solution of the problem* (1)$'$, (2) *is such that at every point* $x \in R^3$ *we have*

$$\lim_{t \to \infty} u(x, t) = 0.$$

iii) *Every solution of the equation* $(-\Delta + q)u = 0$, *satisfying the conditions* $u = O(|x|^{-1})$, $\dfrac{\partial u}{\partial x_k} = O(|x|^{-2})$ *at infinity is identically zero*