246. A Note on Multipliers of Ideals in Function Algebras

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Let X be a compact Hausdorff space and let $C(X)$ be the algebra of all complex-valued continuous functions on X. By a function algebra we mean a closed (by supremum norm) subalgebra in $C(X)$ containing constants and separating points of X. Recently J. Wells [7] has obtained interesting theorems on multipliers of ideals in function algebras. And especially in the disc algebra A_1 it was shown that for any non-zero closed ideal J in A, $\mathfrak{M}_1(J)$ is the set of all H^∞-functions continuous on $D\sim F$, where D is the closed unit disc on the complex plane and F is the intersection of the zeros of the functions in J on the unit circle C ([7], Theorem 8). As A_1 is an essential maximal algebra, the question naturally arises: Does a similar theorem hold for arbitrary essential maximal algebra? The main purpose of this note is to answer the question under certain conditions and to give a generalization of the theorem mentioned above (cf. Theorem 2).

1. Let A be a function algebra on a compact Hausdorff space X. Let J be a non-zero closed ideal in A. By a *multiplier* of J we mean a function φ on $X\sim h(J)$ such that $\varphi J\subset J$, where $h(J)$, the hull of J, is the set of points at which every function in J vanishes. Every multiplier of J is a bounded continuous function on the locally compact space $X\sim h(J)$. We denote the set of all multipliers of J by $\underline{\mathfrak{M}}(J)$. $M(X)$ denotes the set of all complex, finite, regular Borel measures μ on X and a $\mu(\in M(X))$ is orthogonal to A ($\mu\perp A$) means $\int f d\mu=0$ for any $f\in A$. For μ in $M(X)$, μ_F denotes the restriction of μ to F. $C(Y)_\beta$ denotes the space of bounded continuous functions on the locally compact space Y under the strict topology β of Buck ([3], [7]). Let A be a function algebra on X and let F be a closed subset of X. Then F is said to have the *condition* (P) if $\mu_F\perp A$ for every $\mu\perp A$. If F has (P), it is an intersection of peak sets ([4]). Wells [7] has proved the following theorem: $\underline{\mathfrak{M}}(kF)$ is the closure of kF in $C(X\sim F)_\beta$ if and only if F has (P), where $kF=\{f\in A: f(F)=0\}$. Let $F_0=h(J)$, then $\underline{\mathfrak{M}}(kF_0, J)$ denotes the set of all functions φ on $X\sim F_0$ such that $\varphi\cdot kF_0\subset J$. Every function in $\underline{\mathfrak{M}}(kF_0, J)$ is a bounded continuous