245. Some Applications of the Functional-Representations of Normal Operators in Hilbert Spaces. XXV

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In this paper, by applying the functional-representations of normal operators in Hilbert spaces to somewhat abstracted and generalized integral equations, we shall illustrate that the expansions of solutions of such integral equations can be discussed by using integral operators alone even if we do not give any analytic condition from which the expansions of their corresponding kernels can be deduced.

Definitions of notations. Let Δ be a Lebesgue σ-measurable set of finite or infinite measure in real m-dimensional Euclidean space R_m; let $L_2(\Delta, \sigma)$ be the Lebesgue functionspace; let $\{\varphi_\nu(x)\}_{\nu=1,2,3,\dots}$ and $\{\psi_\mu(x)\}_{\mu=1,2,3,\dots}$ be both incomplete orthonormal systems such that the union of them forms a complete orthonormal system in $L_2(\Delta, \sigma)$; let (β_{ij}) be an infinite bounded normal matrix with $\sum_{j=1}^{\infty} |\beta_{ij}|^2 \neq |\beta_{ii}|^2 > 0$ $(i=1, 2, 3, \cdots)$; let $(\beta_{ij}^{(p)}) = (\beta_{ij})^p$ $(p=1, 2, 3, \cdots, n)$ where $\beta_{ij}^{(1)} = \beta_{ij}$ $(i, j=1, 2, 3, \cdots)$; let $\{\lambda_\nu\}_{\nu=1,2,3,\dots}$ be any infinite bounded sequence of complex scalars; and for any positive integer p with $1 \leqq p \leqq n$ and $h(x) \in L_2(\Delta, \sigma)$ let N_p be an integral operator defined by

$$N_p h(x) = \sum_{\nu=1}^{\infty} \lambda_\nu^p \int_\Delta h(y)\overline{\varphi_\nu(y)} d\sigma(y) \cdot \varphi_\nu(x) + c^p \sum_{\mu=1}^{\infty} \left\{ \int_\Delta h(y)\overline{\psi_\mu(y)} d\sigma(y) \cdot \sum_{j=1}^{\infty} \beta_{\mu j}^{(p)} \psi_j(x) \right\},$$

where c is an arbitrarily given complex constant.

Theorem 68. *Let $g(x)$ be an arbitrarily given function in the subspace $\mathfrak{M}$ determined by $\{\varphi_\nu(x)\}_{\nu=1,2,3,\dots}$, and let ζ_p $(p=1, 2, 3, \cdots, n)$ be the roots of the equation $\lambda^n + \sum_{p=1}^{n} a_p \lambda^{n-p} = 0$ with complex coefficients a_p. Then the integral equation*

$$\text{(A)} \qquad \lambda^n f(x) + \sum_{p=1}^{n} a_p \lambda^{n-p} N_p f(x) = g(x) \qquad \left(\lambda \notin \bigcup_{p=1}^{n} \overline{\{\zeta_p \lambda_\nu\}_{\nu=1,2,3,\dots}} \right)$$

has a uniquely determined solution

$$f_\lambda(x) = \sum_{\nu=1}^{\infty} c_\nu \prod_{p=1}^{n} (\lambda - \zeta_p \lambda_\nu)^{-1} \varphi_\nu(x) \in \mathfrak{M} \qquad (\textit{for almost every } x \in \Delta),$$

where $c_\nu = \int_\Delta g(x)\overline{\varphi_\nu(x)} d\sigma(x)$ $(\nu=1, 2, 3, \cdots)$ and $\sum_{\nu=1}^{\infty} |c_\nu|^2 < \infty$; and in addition, if the set $\{\lambda_\nu\}$ is everywhere dense on an open or a closed