244. Representations of Linear Continuous Functionals on the Space C(X, Y) of Continuous Functions from Compact X into Locally Convex Y*)

By Witold M. BOGDANOWICZ

The Catholic University of America, Washington, D. C., U.S.A.

(Comm. by Kinjirô KUNUGI, M.J.A., Dec. 12, 1966)

Let R be the space of reals and Y, Z, W be real Banach spaces. Denote by U the space of all bilinear continuous operators u from the space $Y\times Z$ into W. Norms of elements in the spaces Y, Y', Z, W, U will be denoted by $|\ \ |$.

A nonempty family of sets V of an abstract space X is called a prering if the following conditions are satisfied: (a) if $A_1, A_2\in V$, then $A_1\cap A_2\in V$, (b) if $A_1, A_2\in V$, then there exist disjoint sets $B_1, \cdots, B_k\in V$ such that $A_1\backslash A_2=B_1\cup\cdots\cup B_k$. A function μ from a prering V into a Banach space Z is called a volume if it satisfies the following condition: for every countable family of disjoint sets $A_t\in V(t\in T)$ such that (c) $A=\bigcup_T A_t\in V$, we have $\mu(A)=\sum_T\mu(A_t)$, where the last sum is convergent absolutely and $|\mu|(A)=\sup\{\sum|\mu(A_t)|\}<\infty$ for any $A\in V$, where the supremum is taken over all possible decompositions of the set A into the form (c).

A volume is called positive if it takes on only nonnegative values. If μ is a volume, then its variation $|\mu|$ is a positive volume.

The triple (X, V, v), where v is a fixed positive volume will be called a volume space.

In [1] has been developed the theory of the integral of the form $\int u(f, d\mu)$ defined for $f\in L(|\mu|, Y)$ and $u\in U$, and the theory of the space $L(|\mu|, Y)$ of Lebesgue-Bochner summable functions. For the case of locally compact Hausdorff spaces this integral essentially generates the same operator on the space of summable functions as the integral developed in a different way by Bourbaki [4], Ch. VI, p. 48–49.

The main advantage of the construction of the Lebesgue-Bochner-Stieltjes integration presented in [1] is that one may integrate with respect to any volume μ defined on a prering V without extending it first to a measure.

In this paper by the integral $\int u(f, d\mu)$ we shall understand the integral developed in [1] considered on the space $L(|\mu|, Y)$.

---

*) This research was supported by NSF grant GP 2565.