243. On Julia's Exceptional Functions

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1. Let $f(z)$ be a transcendental meromorphic function and $\rho(f(z))$ the spherical derivative of $f(z)$. O. Lehto and K. I. Virtanen ([2]) have proved that $f(z)$ satisfies

$$(*) \qquad \rho(f(z))=O(1/|z|) \qquad (z \to \infty)$$

if and only if $f(z)$ is a Julia's exceptional function.

Recently, J. M. Anderson and J. Clunie ([1]) have raised an open question whether a function satisfying the condition $(*)$ can possess a Valiron deficient value. We give a negative answer for it in this paper.

2. Let $f(z)$ be a transcendental meromorphic function and $\{\sigma_\nu\}$ an arbitrary sequence of complex numbers such that $|\sigma_\nu|\to\infty$ as $\nu\to\infty$ and $|\sigma_\nu|\geqq 1$. We put $f_\nu(z)=f(\sigma_\nu z)$. If, for every such $\{\sigma_\nu\}$, the family $\{f_\nu(z)\}$ is normal in the sense of Montel in $0<|z|<\infty$, $f(z)$ is said to be a Julia's exceptional function (c.f. A. Ostrowski [4]).

We shall assume the acquaintance with the standard terminology of the Nevanlinna theory:

$$T(r,f)\,, \qquad n(r,a,f)\,, \qquad N(r,a,f)\,, \qquad m(r,a,f)$$

and with the first fundamental theorem of R. Nevanlinna ([3]). The deficiencies of Nevanlinna $\underline{\delta}(a,f)$ and of Valiron $\bar{\delta}(a,f)$ of a value a are defined respectively as follows:

$$\underline{\delta}(a,f)=\varliminf_{r\to\infty}\frac{m(r,a,f)}{T(r,f)}$$

and

$$\bar{\delta}(a,f)=\varlimsup_{r\to\infty}\frac{m(r,a,f)}{T(r,f)}\,.$$

If $\underline{\delta}(a,f)>0$ $(\bar{\delta}(a,f)>0)$, the value a is said to be a Nevanlinna (Valiron) deficient value.

Theorem. *If $f(z)$ is a Julia's exceptional function, then for every complex number a $\bar{\delta}(a,f)=0$.*

Proof. A. Ostrowski ([4]) has proved that if $f(z)$ is a Julia's exceptional function, there exists a finite number C independent of r such that

$$|n(r,0,f)-n(r,\infty,f)|<C.$$

Hence we have

$$n(r,\infty,f)-C<n(r,0,f)<n(r,\infty,f)+C,$$