9. An Algebraic Formulation of K-N Propositional Calculus. II

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In his paper [1], K. Iséki has defined K-N algebra as follows: Let X be an abstract algebra consisting of $0, p, q, \cdots$, with a binary operation $*$ and a unary operation $\sim$ satisfying the following conditions:

a) $\sim(p*p)*p=0$,

b) $\sim p*(q*p)=0$,

c) $\sim\sim(\sim\sim(p*r)*\sim(r*q))*\sim(\sim q*p)=0$,

d) $\sim\sim\beta*\sim\alpha=0$ and $\alpha=0$ imply $\beta=0$, where α, β are expressions in X.

In this paper, we shall show that the NK-algebra is characterized by the following conditions:

1) $\sim(p*p)*p=0$,

2) $\sim q*(q*p)=0$,

3) $\sim\sim(\sim\sim(p*r)*\sim(r*q))*\sim(\sim q*p)=0$,

4) $\sim\sim\beta*\sim\alpha=0$ and $\alpha=0$ imply $\beta=0$, where α, β are expressions (For the details on N-K propositional calculus, see [2], [3], [4].)

K. Iséki has proved that the NK-algebra implies $\sim q*(q*p)=0$. Therefore we shall prove that 1), 2), 3), and 4) imply b).

A) $\sim\alpha*\beta=0$ implies $\sim\sim(\beta*\gamma)*\sim(\gamma*\alpha)=0$.

Proof. In 3), put $p=\beta, q=\alpha, r=\gamma$, then by 4) we have A). Then we have

B) $\sim\alpha*\beta=0, \sim\gamma*\alpha=0$ imply $\beta*\sim\gamma=0$.

In A), put $\alpha=p*p, \beta=p, \gamma=\sim p$, then $\sim(p*p)*p=0$ implies $\sim\sim(p*\sim p)*\sim(\sim p*(p*p))=0$.

By 2), we have

5) $p*\sim p=0$.

In 3), put $p=\sim\sim q, r=\sim r$, then

$$\sim\sim(\sim\sim(\sim\sim q*\sim r)*\sim(\sim r*q))*\sim(\sim q*\sim\sim q)=0.$$

And In 3), put $p=\sim\sim q$, then

$$\sim\sim(\sim\sim(\sim\sim q*r)*\sim(r*q))*\sim(\sim q*\sim\sim q)=0.$$

By 5), $\sim q*\sim\sim q=0$, hence we have

6_1) $\sim\sim(\sim\sim q*\sim r)*\sim(\sim r*q)=0$,

and

6_2) $\sim\sim(\sim\sim q*r)*\sim(r*q)=0$.