2. *Weak Topologies and Injective Modules*

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Wu showed in his paper [1] that a characterization of self-injective rings can be given in terms of weak topologies. The aim of this paper is to generalize this result and give a characterization of injective modules.

Throughout this paper, R will denote a ring (not necessarily commutative) with the identity 1. All R-modules considered will be unitary. We shall show that, if an R-module Q has the property: $\operatorname{ann} Q=0$, where $\operatorname{ann} Q$ denotes the ideal of R consisting of all elements annihilating Q, then a necessary and sufficient condition for the module Q to be injective can be given in terms of weak topologies. In addition to it, we shall show at the end of this paper, that such a simple generalization of the theorem due to Wu is not always available in case $\operatorname{ann} Q\neq 0$.

1. Weak topologies. Let Q be a left R-module. Then $\operatorname{Hom}_R(Q, Q)$ can be regarded as a ring with the identity ι_Q. Λ will denote this ring $\operatorname{Hom}_R(Q, Q)$.

Let M be a left R-module. Then $\operatorname{Hom}_R(M, Q)$ can be regarded as a left Λ-module, since we have $\varphi\circ\rho\in\operatorname{Hom}_R(M, Q)$ for any $\varphi\in\Lambda$ and for any $\rho\in\operatorname{Hom}_R(M, Q)$.

Now we shall give the module Q a structure of topological space with the discrete topology. In connexion with this topology, we shall give the following definition of B-topology on a module M. (cf. Chase [2])

Definition. Let B be a Λ-submodule of the left Λ-module $\operatorname{Hom}_R(M, Q)$. Then the coarsest topology on M such that every element of B is a continuous mapping from M into Q will be called the *weak topology on M induced by B* or simply the *B-topology on M*.

It is easy to see that all subsets of M of the form $\bigcap_{i=1}^{n}\operatorname{Ker}\beta_i$, $\beta_i\in B$, $i=1, 2, \cdots, n$ make a base of neighbourhood system of $0(\in M)$ in the B-topology.

It is obvious that the B-topology on M is Hausdorff if and only if, for each non-zero $x(x\in M)$, there exists $\beta(\beta\in B)$ such that $x\notin\operatorname{Ker}\beta$. According to Wu, we shall say B is *separating* if the weak topology on M induced by B is Hausdorff.

It is evident that $\operatorname{Hom}_R(Q, Q)$-topology on Q is the discrete