1. On the Decomposition of Regular Representation of the Lorentz Group on a Hyperboloid of one Sheet

By Takuro SHINTANI
Department of Mathematics, University of Tokyo

(Comm. by Zyoiti SUETUNA, M.J.A., Jan. 12, 1967)

1. Let G_n be the subgroup of $GL(n+1, R)$ consisting of elements which leave invariant the quadratic form $-x_0^2+x_1^2+\cdots+x_n^2$, and G_n^+ be the connected component of G_n. Let X be the hyperboloid of one sheet in R^{n+1} with the equation $x_0^2-x_1^2-\cdots-x_n^2=-1$. G_n naturally operates on X and the measure on X defined by $dx=\dfrac{dx_1\cdots dx_n}{|x_0|}$ is invariant under the action of G_n. Let $L^2(X)$ be the Hilbert space of functions on X which are square integrable with respect to this measure. Then we get the unitary representation π of G_n^+ on $L^2(X)$ defined as follows: $(\pi(g)f)(x)=f(xg)$, $g\in G_n^+$, $f\in L^2(X)$, $x\in X$. We denote the corresponding representation of the universal enveloping algebra of Lie algebra of G_n^+ also by π. In this note we decompose π into direct sum of irreducible representations. In the following, we use the notations defined in R. Takahashi [1] Chap. I, §1, §2 without further reference.

2. For any complex number s we define the representations $(U^s, \mathcal{H})$ of G_n^+ as follows:

Let H be the linear space of C^∞ functions on K which are invariant under left translations of M and $(U^s(g)f)(k)=e^{-st(k,g)}f(kg)$, $f\in\mathcal{H}$, where kg and $t(k, g)$ is defined uniquely by the relations $kg=a_{t(k,g)}n\,kg$, $a_{t(k,g)}\in A$, $n\in N$, and $kg\in K$. In the following for special value of s we define the positive (in general not definite) inner product $(\ ,\ )_s$ in $\mathcal{H}$ so that U_s becomes unitary, and we get unitary representation $(U_s, \mathcal{H}_s)$ where $\mathcal{H}_s$ is the completion of $\mathcal{H}$ with respect to the norm $\|\ \ \|_s$ defined by inner product $(\ ,\ )_s$.

When $s=-\dfrac{n-1}{2}+i\rho$, $\rho\in R$, we define for any $\varphi, \psi\in\mathcal{H}$, $(\varphi, \psi)_s=\int\varphi\bar{\psi}\,dk$ where dk is the normalized Haar measure of K. For any $\varphi, \psi\in\mathcal{H}$, and s, $\left(Re\,s<-\dfrac{n-1}{2}\right)$ we put

$$I_s(\varphi, \psi)=C_s\int\langle vk_1, vk_2\rangle^{-(n-1+s)}\varphi(k_1)\overline{\psi(k_2)}\,dk_1\,dk_2$$

where $C_s=\dfrac{\sqrt{\pi}\,\Gamma(-s)}{2^{-(1+s)}\Gamma\left(\dfrac{n}{2}\right)\Gamma\left(-\left(s+\dfrac{n-1}{2}\right)\right)}$ and $v=(1, -1, 0, \cdots, 0)$.