31. A Note on Regularity of Null Solutions

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The object of this note is to show that every null solution of partial differential operators of a certain class belongs to the Gevrey class $G_x(0)$ with respect to the space variable. This gives a partial answer for Kumano-go's problem: "Is it possible to construct a null solution such that its derivative of some order has the discontinuity with respect to space-variables at some point (t_0, x_0)?" H. Kumano-go [1].

Our results are stated in the following

Theorem. *Let $L(\lambda, \zeta)$ be a polynomial in λ and ζ with constant coefficients and have the form*

$$(1)\qquad L(\lambda, \zeta)=\sum_{\substack{0\le j\le j_0\\ 0\le k\le k_0}} a_{j,k}\lambda^j\zeta^k,\qquad j_0>0,\ k_0>0,\ \text{and}\ a_{j_0,k_0}=1.$$

Let u be a distribution solution of the equation

$$(2)\qquad L\Big(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}\Big)u=0$$

in R^2. If u vanish when $t\le 0$ and if there exist an open neighbourhood of the t-axis where $(\partial^k/\partial x^k)u$ for $0\le k\le k_0-1$ be functions which are continuously differentiable with respect to t for j_0 times.

Then u is a continuous function of t and x which is entire with respect to x, and satisfies the following inequality

$$(3)\qquad \Big|\frac{\partial^k}{\partial x^k}u(t, x)\Big|\leqq C_0^{k+1}e^{c_1|x|},\quad k=0, 1, 2, \cdots,\quad t\leqq T,$$

where C_0 and C_1 are constants which are independent of k and x.

Remark 1. The telegraph equation

$$u_{tt}=u_{xx}-r^2u\quad (r=\text{constant})$$

can easily be transformed into an operator of the form (1) by introducing new independent variables

$$\xi=t+x,\ \eta=t-x.$$

Remark 2. Let $P(\lambda, \zeta)$ be a polynomial in λ and ζ, and its degree with respect to ζ be equal to $K>0$. We can then write

$$(4)\qquad P(\lambda, \zeta)=Q_0(\lambda)\prod_{k=1}^{K}(\zeta-\zeta_k(\lambda)),$$

where every ζ_k for some positive integer p_k is an analytic function of λ^{-1/p_k} when $|\lambda|>C$, with no essential singularity at infinity, that is,

$$(5)\qquad \zeta_k(\lambda)=\sum_{n=N_k}^{\infty}a_n(\lambda^{-1/p_k})^n.$$