30. On Function Spaces over a Topological Semifield

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This paper is devoted to study function spaces, the elements of the space considered are functions from a topological semifield E to the real number field R.

The purpose of this paper is to introduce a topology into the function space and to consider about the property of continuous functions (see [1]).

Let E be a topological semifield and K the positive part of the semifield E.

Every semifield E is a linear topological space over the real number field R (see [2], [3], and [4]). The set K is a convex cone and its closure $\bar{K}$ is also a convex cone. The cone K is called the positive cone of the topological semifield E.

The set $K^\star$ of all linear functionals which are non-negative on the positive cone K is called the dual cone, and the set $K^\star - K^\star$ considering of all differences of elements of $K^\star$ is the order dual $E^\star$ of E. The cone $K^\star$ defines an order on $E^\star$ which is called the dual ordering of E.

In particular, the topological semifield E satisfies the equality $K-K=E$. Therefore, the dual ordering is anti-symmetric i.e. if $f \geqslant g$ and $g \geqslant f$ then $f=g$. In this case $f-g$ is zero on each element of K and hence on $E=K-K$.

Next we shall consider the condition under which linear functional is the difference of two positive functionals on the topological semifield E.

Proposition 1. *Let E be a topological semifield. For any two elements $x, y \in E$ we set $\rho(x, y)=|x-y|$. The mapping obtained $\rho: E \times E \rightarrow \bar{K}$ transforms E into a metric space over the semifield E. The weak topology of this metric space coincides with the topology of the semifield E.*

Proposition 2. *Let E be a topological semifield. For any $x \in E$ we set $\|x\|=|x|$. Then E becomes a normed space over the semifield E. The weak topology of this normed space coincides with the topology of the semifield E.*

Let E be a normed space and let E^* be the space of all continuous real-valued linear function on E. The norm topology for the adjoint space E^* is defined by $\|f\|=\sup\{|f(x)|:\|x\|<1\}$. The topology of