29. *An Algebraic Formulation of K-N Propositional Calculus. II*

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In his paper [1], K. Iséki defined the *NK-algebra*. For the details of the *NK-algebra*, see [1]. The conditions of the *NK-algebra* are as follows:

a) $\sim(p*p)*p=0$,

b) $\sim p*(q*p)=0$,

c) $\sim\sim(\sim\sim(p*r)*\sim(r*q))*\sim(\sim q*p)=0$,

d) Let α, β be expressions in this system, then

$$\sim\sim\beta*\sim\alpha=0 \text{ and } \alpha=0 \textit{ imply } \beta=0.$$

In this note, we shall show that a *NK-algebra* is implied by the following conditions:

1) $\sim(p*p)*p=0$,

2) $p*(\sim p*q)=0$,

3) $\sim\sim(\sim\sim(p*r)*\sim(r*q))*\sim(\sim q*p)=0$,

4) $\sim\sim\beta*\sim\alpha=0$ and $\alpha=0$ imply $\beta=0$, where α, β are expressions in this system. We shall prove that 1)–4) imply b)

In 3), put $p=\beta, q=\alpha, r=\gamma$, then

$$\sim\sim(\sim\sim(\beta*\gamma)*\sim(\gamma*\alpha))*\sim(\sim\alpha*\beta)=0.$$

By 4), we have $\sim\sim(\beta*\gamma)*\sim(\gamma*\alpha)=0$. Then we have the followings:

A) $\sim\alpha*\beta=0$ *implies* $\sim\sim(\beta*\gamma)*\sim(\gamma*\alpha)=0$,

B) $\sim\alpha*\beta=0, \gamma*\alpha=0$ *imply* $\beta*\gamma=0$,

C) $\sim\alpha*\beta=0, \sim\gamma*\alpha=0$ *imply* $\beta*\sim\gamma=0$.

In B), put $\alpha=\sim p*\sim p, \beta=\sim p, \gamma=p$, then

$$\sim(\sim p*\sim p)*\sim p=0,\ p*(\sim p*\sim p)=0 \text{ imply } \sim p*p=0.$$

By 1) and 2) we have

5) $\sim p*p=0$.

In 3), put $q=p$, then

$$\sim\sim(\sim\sim(p*r)*\sim(r*p))*\sim(\sim p*p)=0.$$

By 5) we have

6) $\sim\sim(p*r)*\sim(r*p)=0$.

In 3), put $q=\sim p, p=\sim p, r=\sim\sim p$, then

$$\sim\sim(\sim\sim(\sim p*\sim\sim p)*\sim(\sim\sim p*\sim p))*\sim(\sim\sim p*\sim p)=0.$$

By 5), we have

7) $\sim p*\sim\sim p=0$.

In 6), put $p=\alpha, r=\beta$, then $\sim\sim(\alpha*\beta)*\sim(\beta*\alpha)=0$ implies $\alpha*\beta=0$. Hence by 4) we have