

## 28. Axiom Systems of Aristotle Traditional Logic

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In this paper, we shall consider axiom systems of Aristotle traditional logic. Some axiom systems are obtained by J. Lukasiewicz ([3], [4]), I. Bocheński [1] and N. Kretzmann [2]. We shall give a method to find axiom systems. The method given below is useful for the purpose, and all axiom systems are found by our method.

As well known (see A. N. Prior [5]), in the Aristotle traditional logic, there are four types of categorical propositions,  $a$  and  $b$  being terms:

- 1) Every  $a$  is  $b$ .
- 2) At least one  $a$  is  $b$ .
- 3) At least one  $a$  is not  $b$ .
- 4) No  $a$  is  $b$ .

These propositions are denoted by  $Aab$ ,  $Iab$ ,  $Oab$ ,  $Eab$  respectively. Let  $N$  be the negation functor, then  $Eab = NIab$ ,  $Oab = NAab$ . By  $X, Y, Z$ , we denote the term functors  $A, I, O, E$ , then there are two different kinds of moods: *immediate inference* and *sylogism*. Moreover, the mood of immediate inference is divided into two types:  $CXabYab$ ,  $CXabYba$ , where  $C$  is the implication functor. These are denoted by  $XY_1$ ,  $XY_2$  respectively. On the syllogism, we have four types of moods:

- I)  $CKXabYcaZcb$ ,
- II)  $CKXabYcbZca$ ,
- III)  $CKXabYacZcb$ ,
- IV)  $CKXabYbcZca$ ,

where  $K$  is the conjunction functor. These moods are denoted by  $XYZ_i$  ( $i=1, 2, 3, 4$ ) respectively.

In these symbols, the Lukasiewicz axiom system is written in the form of

- L1)  $Aaa$ ,
- L2)  $Iaa$ ,
- L3)  $AAA_1$ ,
- L4)  $AII_3$ .

Assuming some theses of propositional calculus, we have  $II_1$ ,  $II_2$ ,  $EE_1$ ,  $EE_2$ ,  $AI_1$ ,  $AI_2$ ,  $EO_1$ ,  $EO_2$  (see J. Lukasiewicz [3]).

From theses of the classical propositional calculus, we have the following deduction rules: