27. *Concerning Paracompact Spaces*[*]

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Tukey [8] investigated spaces with certain property of the refinements of their open covers, called fully normal spaces, while Dieudonné's generalized compact spaces are those having locally finite refinements for the open covers. Fully normal spaces and paracompact spaces were shown to be the same by A. H. Stone [7]. Compact Hausdorff spaces are characterized by one of the equivalent properties: (1) Every open cover of the space of power $\geq\aleph_0$ (the cardinal number of the set of all positive integers) has a subcover of power $<\aleph_0$. (2) Every net in the space has a cluster point, and (3) Every ultrafiler converges to a point. The question arises, how do these properties of compact spaces reappear in paracompact spaces? [4, p. 208]. Corson [1] gave a characterization of paracompact spaces analoguous to the property (3) by showing that a space is paracompact if and only if every Cauchy-like ultrafilter converges to a point. Whether there exist the analogies of the first two properties remains an open problem, that is, "Is there a cardinal number $\aleph$ associated to a space such that the space is paracompact if and only if every open cover of power $\geq\aleph$ has a subcover of power $<\aleph$?" and "Is there a class of nets with the property: the paracompactness of the space is equivalent to the existence of a cluster point of each net in the class?" Theorem 1 in this note will give affirmative answers to the questions.

Deudonné [2] showed that the cartesian product of a paracompact space and a compact space is paracompact and Michael's [5] sharpened result is that the compactness of one of the coordinate spaces can be replaced by σ-compactness. The general statement, relative to the paracompactness of the product of two paracompact spaces, has been ruled out by Sorgenfrey's counter-example [6]. We will show in Theorem 2 that the product of a paracompact space and a locally compact paracompact space is paracompact.

Definition. Let $\{x_\delta\,;\,\delta\in D\}$ be a net. The family of the cardinal numbers of all cofinal subsets of $\{x_\delta\}$ contains a smallest number $\aleph$ which is called the least cardinal number of $\{x_\delta\}$.

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