25. On the Crossed Product of Abelian von Neumann Algebras. I

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1. H. A. Dye has successfully investigated in detail the groups of measure preserving transformations on a finite measure space in his deep studies [3] and [4]. Among many others, Dye has introduced the notion "equivalence" among these groups. In the present note, we shall discuss his notion in connection with the crossed product of an abelian von Neumann algebra.

2. Throughout the note, we shall use the terminology on von Neumann algebras due to J. Dixmier [2] without further explanations.

Following after Dye [3], we shall introduce some fundamental definitions on automorphisms of an abelian von Neumann algebra $\mathcal{A}$ with the faithful normal trace ϕ normalized by $\phi(1)=1$. A projection P in $\mathcal{A}$ is said to be *absolutely fixed* under an automorphism g of $\mathcal{A}$ if $Q^g=Q$ for each $Q\leqq P$. For the given two automorphisms g and h of $\mathcal{A}$, we shall denote by $F(g, h)$ the maximal projection in $\mathcal{A}$ which is absolutely fixed under gh^{-1}.

Let G be a group of automorphisms of $\mathcal{A}$ which is ϕ-preserving in the sense that $\phi(A^g)=\phi(A)$ for each $A\in\mathcal{A}$ and $g\in G$. If $F(g, 1)=0$ for each $g\neq 1$ in G, then G is called *freely acting*. If α is an automorphism of $\mathcal{A}$, we say that α *depends* on G if $\text{l.u.b.}_{g\in G} F(\alpha, g)=1$. We shall denote $[G]$ by the collection of all automorphisms of $\mathcal{A}$ which preserve ϕ and depend on G. Two groups of G_1 and G_2 of ϕ-preserving automorphisms of $\mathcal{A}$ will be called *equivalent*, if they determine the same full group, that is, $[G_1]=[G_2]$.

3. At first we shall review briefly the concept of the crossed product of an abelian von Neumann algebra $\mathcal{A}$ with the faithful normal trace ϕ normalized with $\phi(1)=1$ by an enumerable freely acting group G of ϕ-preserving automorphisms of $\mathcal{A}$, cf. for instance [5] and [6].

We shall denote an operator valued function defined on G by $\sum_{g\in G} g\otimes A_g$ where $A_g\in\mathcal{A}$ is the value of the function at $g\in G$. Let $\mathcal{D}$ be the set of all functions such that $A_g=0$ up to a finite subset of G. Then $\mathcal{D}$ is a linear space with the usual operations