# 22. On a Sum Theorem in Dimension Theory

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The present paper deals primarily with the sum theorems for the large inductive dimension of totally normal spaces.[1] In this connection C. H. Dowker established in [2] a sum theorem which is stated as follows: *Let* $\{A_i\}$ *be a countable number of closed sets in a totally normal space and let* $\operatorname{Ind} A_i$[2] $\leqq n, i=1, 2, \cdots$. *Then* $\operatorname{Ind}\left(\bigcup_{i=1}^{\infty} A_i\right) \leqq n$. Corresponding to this result, we established in [3] the following theorem. *Let* $\{A_\alpha \mid \alpha<\Omega\}$ *be a locally finite closed covering of a totally normal and countably paracompact space* $X$ *and let* $\operatorname{Ind} A_\alpha \leqq n$ *for each* $\alpha$. *Then* $\operatorname{Ind} X \leqq n$.

Our present object is to show that the countable paracompactness condition in the above theorem is redundant. Indeed, our main theorem reads as follows: *Let* $\{A_\alpha \mid \alpha<\Omega\}$ *be a locally finite collection of closed sets in a totally normal space* $X$ *and let* $\operatorname{Ind} A_\alpha \leqq n$ *for each* $\alpha$. *Then* $\operatorname{Ind}\left(\bigcup_{\alpha<\Omega} A_\alpha\right) \leqq n$. For the proof of our theorems we shall need some of Dowker's results.

**1. Preliminary theorems due to C. H. Dowker.** A normal space $X$ is called *totally normal* ([2, § 4]) if each open set $G$ is the union of a collection $\{G_\alpha\}$, locally finite in $G$, of open $F_\sigma$ sets of $X$. The following theorems are due to C. H. Dowker and they form the basis of a proof for our theorems.

**Theorem 1.** ([2, 4.1], [2, 4.2], and [2, 4.6]). *Every perfectly normal space or every hereditarily paracompact space is totally normal and every totally normal space is completely normal.*[3]

The converse of Theorem 1 is not true as is observed by the well-known Bing's examples ([1]).

**Theorem 2.** ([2, 4.7]). *The total normality is hereditary; that is, every subspace of a totally normal space is also totally normal.*

**Theorem 3.** ([2, Theorem 2]). *In a totally normal space* $X$ *let* $A \subset X$. *Then* $\operatorname{Ind} A \leqq \operatorname{Ind} X$.

Theorem 3 is referred to as "the subset theorem".

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1) Throughout the paper by a *space* we mean a $T_1$-space.

2) Ind $X$ means the *large inductive dimension* of a space $X$ defined inductively in terms of closed sets. For a detailed definition, see [2].

3) Some authors refer to "completely normal" as "hereditarily normal" (e.g. [5]).