19. *Concrete Characterization of the Domains of Fractional Powers of Some Elliptic Differential Operators of the Second Order*

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1. **Introduction.** Recently J. L. Lions has succeeded in characterizing the domains of the fractional powers of an arbitrary regularly accretive operator[1] A in a Hilbert space H as interpolation spaces between the domain of the operator and the whole Hilbert space H.[2] In this connection it would be worth while to characterize domains of fractional powers of differential operators as concretely as possible. However it seems that little is known in this direction.

In the present note we shall be concerned with the following problem: Let Ω be a bounded domain in $\boldsymbol{R}^m$ with the boundary $\partial\Omega$ which is a $m-1$ dimensional C^∞ manifold. The unit outer normal to $\partial\Omega$ is denoted by n. Let A_α be a regularly accretive operator in $L^2(\Omega)$ mapping $D_\alpha=\left\{u\in H^2(\Omega);\ \alpha\dfrac{\partial u}{\partial n}+(1-\alpha)u|_{\partial\Omega}=0\right\}$ into $L^2(\Omega)$, where $H^s(\Omega)$, $s\geq 0$, denotes the Sobolev space of order s and where $\alpha\equiv\alpha(x')$ is a given C^∞ function defined for x' in $\partial\Omega$ with $0\leq\alpha\leq 1$. Then, we ask how one can characterize the domain $D(A_\alpha^{1-\theta})$, $0\leq\theta\leq 1$, of the fractional power $A_\alpha^{1-\theta}$ of A_α. The purpose of the present note is to give a fairly satisfactory concrete answer to this question assuming that on each of connected components of $\partial\Omega$ either α vanishes identically or α is never equal to zero.[3]

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§ 2. **Results.** Since the boundary $\partial\Omega$ is smooth, there is a system of first order differential operators $\tilde{D}_1,\cdots,\tilde{D}_{m-1}$, the restriction of which to the boundary forms a basis of first order differential operators tangential to $\partial\Omega$. $\zeta(x)$ denotes the distance from x in $\boldsymbol{R}^m$ to $\partial\Omega$.

Now we state the result in the case of $\alpha\equiv 0$, that is, the Dirichlet boundary condition.

1) See Definition 2.1 and Theorem 2.1 in [1].
2) See Théorème 3.1 in [2].
3) Detailed proofs of theorems in this note will be published elsewhere.