43. *An Approach to the Theory of Integration Generated by Positive Linear Functionals and Existence of Minimal Extensions*[*),**)]

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A family of sets V of an abstract space X is called a *prering* if V is nonvoid and $A_1, A_2 \in V$ implies $A_1 \cap A_2 \in V$ and there exist disjoint sets $B_1, \cdots, B_k \in V$ such that $A_1 \backslash A_2 = B_1 \cup \cdots \cup B_k$.

A nonnegative function V on a prering is called a *volume* if V is finite-valued and for every countable family $A_t \in V(t \in T)$ of disjoint sets such that $A = \bigcup_T A_t \in V$, we have $v(A) = \sum_T v(A_t)$. Such a triple (X, V, v) will be called a volume space.

In [1] has been presented a direct approach to the theory of Lebesgue-Bochner integration generated by a volume space (X, V, v). The construction of the theory was not based on the theory of measurable functions or on the theory of measure. The construction of the theory of Lebesgue-Bochner measurable functions and of the theory of measure corresponding to this approach has been developed in [3]. In this paper will be presented an approach to the theory of integration generated by a positive linear functional based only on the results of [1].

In § 1 are given equivalent conditions for a linear positive functional on a linear lattice to be a Daniell functional. An extension of the Daniell functional is constructed leading to a positive volume.

In § 2 are given conditions for the integral functional generated by the volume to be an extension of the Daniell functional.

In § 3 are given theorems concerning of the existence of the *smallest extension of a Daniell functional* to an integral functional generated by a volume. It is proven that the volume constructed in § 2 generates the smallest extension, provided that every function $f \in C_0$ is summable with respect to that volume.

In § 4 are given representations of a Daniell functional by means of integral functionals generated by measures. Existence of the smallest measures representing the extensions of the functional are established.

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) The result contained in Theorem 4 of this paper has been presented to the American Mathematical Society, Notices AMS **13 (1966), p. 89.