42. *Integration with Respect to the Generalized Measure. II*

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The purpose of this part of the present paper is to state a proof of Theorem 1 in [1].

Remark. The proof of Theorem 1 in [1] follows from the propositions (except Propositions 3.1 and 3.2) in section 3 in [1] and therefore this theorem also holds if we replace the assumption for $\mathcal{S}$ in the definition of a structure by the condition stated in the remark in section 3 in [1].

Denote by $\mathcal{G}_1$ the perfection of $\mathcal{G}$ and by $\mathcal{G}_2$ the perfection of the closure $\bar{\mathcal{G}}_1$ of $\mathcal{G}_1$ in $\mathcal{F}$.

Lemma 1. *The integral closure $\tilde{\mathcal{G}}$ of $\mathcal{G}$ is the $\mathcal{F}$-completion of $\mathcal{G}_2$.*

Proof. Let $\mathcal{G}_3$ be the $\mathcal{F}$-completion of $\mathcal{G}_2$. Then Proposition 3.10 [1] implies that $\mathcal{G}_3$ is the $\mathcal{F}$-completion of $\bar{\mathcal{G}}_1$. Hence it follows from Proposition 3.17 [1] that $\mathcal{G}_3$ is closed and therefore $\mathcal{G}_3$ is i-closed. To prove that $\mathcal{G}\subset\mathcal{G}_3$, let us consider the $\mathcal{F}$-completion $\mathcal{G}_4$ of $\mathcal{G}_1$. Then Proposition 3.10 [1] implies that $\mathcal{G}\subset\mathcal{G}_4$ and the formula $\mathcal{G}_1\subset\bar{\mathcal{G}}_1$ implies that $\mathcal{G}_4\subset\mathcal{G}_3$. Thus we have $\mathcal{G}\subset\mathcal{G}_3$. It is easily verified that $\mathcal{G}_3$ is the smallest of i-closed subgroups of $\mathcal{F}$ containing $\mathcal{G}$. This proves the lemma.

Let I be the perfection of $\mathcal{I}$ and let I_X be the restriction of I on $X\mathcal{G}_1$ for each $X\in\mathcal{S}$. Then I_X is a continuous homomorphism of $X\mathcal{G}_1$ into J for each $X\in\mathcal{S}$.

Lemma 2. *I_X is uniquely extended to a continuous homomorphism $\bar{I}_X$ of $X\bar{\mathcal{G}}_1$ into J for each $X\in\mathcal{S}$.*

Proof. From the continuity of X, it follows that $X\bar{\mathcal{G}}_1\subset\overline{X\mathcal{G}_1}$ and therefore that $X\mathcal{G}_1$ is dense in $X\bar{\mathcal{G}}_1$. Since J is Hausdorff and complete, this lemma follows from Bourbaki.[1)]

Considering the map $\bar{I}_X$ in Lemma 2, we have

Lemma 3. *There uniquely exists an integral map $\bar{I}$ with respect to $(\mathcal{S}, \mathcal{G}_2, J)$ such that the restriction of $\bar{I}$ on $X\bar{\mathcal{G}}_1$ coincides with $\bar{I}_X$ for each $X\in\mathcal{S}$.*

Proof. Let us prove that $\bar{I}_X(f)=\bar{I}_Y(f)$ for $X, Y\in\mathcal{S}$, and

1) [2] chap. III. Groupes Topologiques, §3, no 3, Proposition 5.