68. *On Extensions of Automorphisms of Abelian von Neumann Algebras*

By Marie CHODA and Hisashi CHODA

Department of Mathematics, Osaka Gakugei Daigaku

(Comm. by Kinjirô KUNUGI, M.J.A., April 12, 1967)

1. Let $\mathcal{A}$ be a maximal abelian von Neumann algebra acting on a separable Hilbert space $\mathfrak{H}$, ϕ a faithful normal trace with a normalized trace vector and G a countable freely acting ergodic group of ϕ-preserving automorphisms of $\mathcal{A}$. Then we can raise the following questions with respect to automorphisms of $\mathcal{A}$ and automorphisms of the crossed product $G\otimes\mathcal{A}$ of $\mathcal{A}$ by G.

1) What kind of automorphisms of $\mathcal{A}$ can be extended to what kind of automorphisms of $G\otimes\mathcal{A}$?

2) Especially, what kind of automorphisms of $\mathcal{A}$ can be extended to inner automorphisms of $G\otimes\mathcal{A}$?

3) What kind of unitary operators in $G\otimes\mathcal{A}$ induce inner automorphisms of $G\otimes\mathcal{A}$ which preserve $\mathcal{A}$?

4) How does the questions 1) or 2) depend on the properties of G?

In this paper, the questions 1) and 4) will be discussed according to several conditions. The questions 2) and 3) are already discussed in [1] and [4].

Hereafter, we assume all automorphisms of $\mathcal{A}$ are ϕ-preserving *-automorphisms, and the terminology and the notations of [2] will be employed without further explanations.

2. We shall reformulate a theorem of I. M. Singer [5; Lemma 2.2] using the terminology of the crossed product:

Theorem 1. *Let $\mathcal{A}$ be a maximal abelian von Neumann algebra acting on a separable Hilbert space $\mathfrak{H}$, ϕ a faithful normal trace with a normalized trace vector, G a countable freely acting ergodic group of automorphisms of $\mathcal{A}$ and σ an inner automorphism of $G\otimes\mathcal{A}$ such that $\mathcal{A}^\sigma=\mathcal{A}$.*

Then σ is induced by a unitary operator

$$U=\sum_{g\in G} VE_gU_g,$$

where V and E_g satisfy the following conditions:

(1) *V is a unitary operator in $\mathcal{A}$,*

(2) *E_g is a projection in $\mathcal{A}$ for each $g\in G$,*

(3) *$E_gE_h=0$ for $g\neq h$,*

(4) *$\sum_{g\in G} E_g=1$,*