59. *Infinite Product of Ergodic Flows with Pure Point Spectra*

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J. von Neumann has shown that two ergodic flows with the same pure point spectrum are mutually metrically equivalent, and that they are isomorphic to the canonical flow on a compact Abelian group. While non-ergodic flows with the same pure point spectrum are not always mutually metrically equivalent.

In this paper, we shall show that, for a certain class of non-ergodic flows with pure point spectra, the spectral type determines the flow metrically. Flows of this class often appear as flows induced by stationary stochastic processes and as transversal flows of automorphisms.

1. Canonical flows on compact Abelian groups. Let G be a separable compact Abelian group, m be the normalized Haar measure of G and $\mathfrak{M}$ be the minimum complete σ-field generated by all open subsets of G. Then $(G, \mathfrak{M}, m)$ is a Lebesgue space in the sense of V. A. Rohlin [4]. Let $\{\alpha^t\}$ be a one-parameter subgroup of G. Then the flow $\{S_t\}$ on G is defined by

(1,1) $$S_t g = \alpha^t g \qquad \text{for } g \in G.$$

Definition 1. We call $\{S_t\}$ the G-flow induced by $\{\alpha^t\}$.

The G-flow $\{S_t\}$ is measurable and it is ergodic if and only if $H = \{\alpha^t; -\infty < t < \infty\}$ is dense in G.

If Λ is a countable subgroup of the additive group R of real numbers, then its character group G is a separable compact Abelian group and there exists a one-parameter subgroup $\{\alpha^t\}$ of G, such that

(1,2) $$\alpha^t(\lambda) = \exp[it\lambda] \qquad \text{for } \lambda \in \Lambda.$$

Let $\{S_t\}$ be the G-flow induced by the $\{\alpha^t\}$. Then $\{S_t\}$ is an ergodic flow with the pure point spectrum Λ. Conversely, for any ergodic measurable flow on a Lebesgue space, the discrete part of the spectrum forms a countable subgroup of R.

Definition 2. We call the G-flow induced by this $\{\alpha^t\}$ a canonical flow on G.

Theorem 1. (*J. von Neumann*). *Let $\{T_t\}$ be an ergodic measurable flow on a Lebesgue space with a pure point spectrum Λ. Let $\{S_t\}$ be a canonical flow on G which is the character group of Λ. Then $\{T_t\}$ and $\{S_t\}$ are isomorphic to each other.*