81. Moments of the Last Exit Times

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(Comm. by Zyoiti SUETUNA, M.J.A., May 12, 1967)

We consider the typical stochastic processes and give the several conditions for the existence of arbitrary order moments of the last exit times from given sets. The answers will vary quite according to the dimension of the state space.

§1. **Stable process.** In this section we will consider a stable process, $X(t)$, on R^d (d-dimensional Euclidean space) having index α, $0<\alpha\leqq 2$, and normalized so that the path functions are right continuous with left hand limits at every point. For simplicity, we restrict our discussion to symmetric stable processes, that is, processes with stationary independent increments having continuous transition density

$$p(t, x, y)=p(t, y-x)=(2\pi)^{-d}\int_{R^d} e^{-i(\theta,y-x)} e^{-t|\theta|^\alpha} d\theta.$$

We will write P_x and E_x for the conditional probability and expectation under the condition $X(0)=x$.

For a bounded Borel (more generally, analytic) set $B\subset R^d$, let

$$\begin{aligned} T_B &= \sup\{t\geqq 0;\ X(t)\in B\} \\ &= 0, \qquad \text{if } X(t)\notin B \text{ for all } t>0, \end{aligned}$$

denote the *last exit time* of B, and let

$$\begin{aligned} V_B &= \inf\{t>0;\ X(t)\in B\} \\ &= \infty, \qquad \text{if } X(t)\notin B \text{ for all } t>0, \end{aligned}$$

be the *first hitting time* into B.

Theorem 1. *In the transient symmetric stable process, namely the case $d>\alpha$, the relation*

$$P_x[T_B\in dt]=\int_{\bar B} p(t, x, y)\mu_B(dy)dt \text{ [2)]}$$

holds. Here μ_B is the equilibrium (capacitary) measure of B.

This theorem is due to S. Watanabe [6]. Recently by the same principle, S. C. Port [2] gave the following result:

$$P_x[T_B>t]=\int_{\bar B}\int_t^\infty p(s, x, y)ds\mu_B(dy)$$

from which our theorem immediately follows.

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1) Financial support from the Sakkokai Foundation for the author is gratefully acknowledged.

2) The notation $\bar B$ means closure of B.