107. *On a Certain Class of Univalent Functions*

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Let us consider a simply connected polygon which has $2n$ sides parallel to the real axis or imaginary axis in the w-plane. If we call its vertices $w_1, w_2, \cdots, w_2$ and denote its interior angles $\pi\alpha_1, \pi\alpha_2, \cdots, \pi\alpha_{2n}$ respectively, α_k takes the value 1/2 or 3/2, and $\sum_{k=1}^{2n}\alpha_k$ is equal to $2n-2$.

We can construct the function $w=f(z)$ which maps the interior of unit circle $|z|<1$ onto the interior of this polygon by

$$\frac{dw}{dz}=K(z-z_1)^{\alpha_1-1}(Z-z_2)^{\alpha_2-1}\cdots(z-z_{2n})^{\alpha_{2n}-1}, \tag{1}$$

where $z_k=e^{i\theta_k}(0\leqq\theta_1<\theta_2<\cdots<\theta_{2n}<2\pi)$ are points on the unit circle $|z|=1$, and k is a constant complex number. The equality (1) is known as Schwarz-Christoffel's formula.

If we put $z_k^{-1}=\varepsilon_k$, we have

$$\frac{dw}{dz}=C(1-\varepsilon_1 z)^{\delta_1}(1-\varepsilon_2 z)^{\delta_2}\cdots(1-\varepsilon_{2n}z)^{\delta_{2n}}, \tag{2}$$

where C is a constant, δ_k is equal to 1/2 or $-1/2$ and $\sum_{k=1}^{2n}\delta_k$ is equal to -2. And square roots in (2) mean to take the branch such that $\sqrt{1}=1$. The function $\dfrac{dw}{dz}$ above defined is analytic for $|z|<1$ and $w=f(z)$ is analytic and univalent for $|z|<1$.

Next we consider a polygon shown in Fig. 1. In this case, we can write signs of δ_k in order and if we take apart suitable four minus signs, we can arrange a sequence of couples $(-+)$ or $(+-)$ as follows,

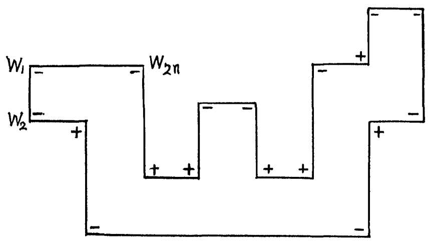

Fig. 1

$$\ominus\ominus(+-)(-+)\ominus\ominus(-+)(-+)(+-)(-+)(+-). \tag{3}$$

We shall denote a class of functions $w=f(z)$ which map the interior of unit circle respectively onto the interior of a polygon which has the nature above mentioned by the symbol S_0. For a function which belongs to the class S_0, we have the following theorem.

Theorem. *Let $w=f(z)$ be a function which belongs to the class S_0, and let*