106. *On Normal Analytic Sets*

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In this paper, we shall show that an irreducible analytic set at a point is always described locally by a certain number of systems of Puiseux-series. And we shall present a theorem saying that an irreducible analytic set in a neighborhood of a point is *normal* if and only if such a group of systems of Puiseux-series satisfies the following conditions.

(1) Two systems of the group never pass through any common point.

(2) If the order of the series of a system belonging to the group exceeds 1, the second coefficient of a series of the system does not vanish identically.

We shall further give a theorem concerning the dimension of the set of non-normal points.

We suppose that the analytic sets are in the space of n complex variables and of d-dimension at the point we consider, where n surpasses 2 and d surpasses 1—the reason of which is that, if $d=1$, all the circumstances reduce to a very clear situation and our results subsist without any alteration.

1. Representation by systems of Puiseux-series. We work in the space of n variables $x_1, \cdots, x_n (n>2)$. Let $\sum$ be an analytic set in an open set containing a point A; we suppose that $\sum$ is *irreducible at* A. For brevity, we assume that A is the origin. Then by the local description theorem,[1] if we choose a proper system of coordinates, there exist a polydisc

$$C: |x_i|<r_i,\ 1\leqslant i\leqslant n,$$

with arbitrarily small radii r_i, a distinguished pseudo-polynomial of degree an integer N in x_{d+1}:

$$P(x_1, \cdots, x_{d+1})=x_{d+1}^N+a_1(x_1, \cdots, x_d)x_{d+1}^{N-1}+ \cdots +a_N(x_1, \cdots, x_d)\ ,$$

and, for each $i, d+1<i\leqslant n$, a pseudo-polynomial of degree $\leqslant N-1$ in x_{d+1}:

$$Q_i(x_1, \cdots, x_{d+1})=b_1^{(i)}(x_1, \cdots, x_d)x_{d+1}^{N-1}+\cdots+b_N^{(i)}(x_1, \cdots, x_d)\ ,$$

such that they together satisfy the following conditions.

(1) The coefficients $a_j(x_1, \cdots, x_d), b_j^{(i)}(x_1, \cdots, x_d)$ of P, Q_i are holomorphic in the polydisc

1) M. Hervé: Several Complex Variables, Tata Institute of Fundamental Research, Bombay, Oxford University Press, 1963.