105. On Certain Condition for the Principle of Limiting Amplitude. II

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1. Introduction and results. We consider the problem

$$(1)\qquad \left[\frac{\partial^2}{\partial t^2} - \Delta + q(x)\right] u(x, t) = 0 \qquad (t>0),$$

$$u(x, 0) = 0, \qquad \frac{\partial u}{\partial t}(x, 0) = f(x),$$

where x is a point of 3-dimensional Euclidean space $E=R^3$, and Δ denotes the Laplace operator in E.

In an earlier paper [1], for the case that q has compact support we proved that under the certain condition the principle of limit amplitude for the problem (1) is valid if and only if there exists no solution $\omega \notin L^2(E)$ of the equation $(-\Delta+q)\omega=0$ satisfying conditions $\omega = O(|x|^{-1})$, $\dfrac{\partial \omega}{\partial x_i} = O(|x|^{-2})$ $(|x|\to\infty)$ (see [2]).

In the present paper we shall prove the same one for the case that the support of q is not compact.

Through the present paper $q(x)$ and $f(x)$ are assumed to satisfy the following conditions (C_1), (C_2), and (C_3):

(C_1) *$q(x)$ is a locally Hölder continuous real-valued function and behaves like $O(|x|^{-2-\alpha})$ $(\alpha>0)$ at infinity.*

By A we denote the unique self-adjoint extension in $L^2(E)$ of $-\Delta+q$ defined on $C_0^\infty(E)$.

(C_2) *A has no eigenvalue.*

Then A is positive definite.

(C_3) *f belongs to the domain $D(A^{\frac{1}{2}})$ of the self-adjoint operator $A^{\frac{1}{2}}$ and behaves like $O(|x|^{-3-\alpha})$ at infinity.*

Under the assumptions (C_1), (C_2), and (C_3) we have the followings:

Theorem 1. *Suppose that $\langle f, \omega\rangle = 0$, where ω is the preceding one and $\langle f, \omega\rangle$ denotes $\int_E f(x)\omega(x)dx$. Then for the solution $u(t) \equiv u(x, t)$ of (1) we have*

$$\lim_{t\to\infty} (u(t), \varphi)_{L^2(E)} = 0 \qquad \textit{for all } \varphi \in L^2(E),$$

and

$$\lim_{t\to\infty} \| u(t) \|_{L^2(K)} = 0 \qquad \textit{for all compact } K \subset E.$$