102. *On Diffeomorphisms of the n-Disk**)

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1. **Introduction.** Let D^n denote the closed unit n-disk in R^n and let Diff (D^n) denote the group of orientation preserving C^1 diffeomorphisms of D^n onto itself. We show here that, under a suitable topology, the injection $\mathrm{SO}(n) \to \mathrm{Diff}(D^n)$ is a weak homotopy equivalence. If follows as a corollary that every orientation preserving diffeomorphism of S^n onto itself which extends to a diffeomorphism of D^n is isotopic to the identity through such diffeomorphisms. This partially answers a question of Smale.

In the last section of the paper, we consider Diff (D^n) in the C^1 topology and show that either $\mathrm{SO}(6) \to \mathrm{Diff}(D^6)$ is not a weak homotopy equivalence or SO (6) is not a deformation retract of Diff (S^5).

2. **Preliminaries.** Suppose $f \in \mathrm{Diff}(D^n)$, $\varepsilon > 0$, and C is a compact subset of the interior of D^n. Let $W(f, \varepsilon, C)$ denote the set of all $g \in \mathrm{Diff}(D^n)$ such that

$$|f(x) - g(x)| < \varepsilon \qquad \text{for all } x \in D^n$$

and

$$|\partial f_i/\partial x_k(x) - \partial g_i/\partial x_k(x)| < \varepsilon \qquad \text{for all } x \in C;\ i, k = 1, \cdots, n.$$

We take the sets $W(f, \varepsilon, C)$ as a basis for our special topology on Diff (D^n).

Let B^n denote the interior of D^n and let Diff (B^n) denote the group of orientation preserving homeomorphisms of B^n in the coarse C^1 topology [6]. Let EDiff (B^n) denote the subset of Diff (B^n) consisting of elements which are extendable to diffeomorphisms of D^n. We endow EDiff (B^n) with the topology it inherits from Diff (B^n). We let EDiff (D^n) denote the set Diff (D^n) with the topology induced from EDiff (B^n) by the inclusion map $i: B^n \to D^n$.

Stewart [9] has shown that SO (n) is a strong deformation retract of Diff (B^n). Since EDiff (B^n) is mapped into itself throughout this deformation retraction, we have that SO (n) is a strong deformation retract of EDiff (B^n) also.

Let EDiff (S^n) denote the set of orientation preserving diffeomorphisms of S^n onto itself which are extendable to diffeomorphisms of D^n. We give EDiff (S^n) the compact-open topology.

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