99. *A Note on Extended Regular Functional Spaces*

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1. Beurling and Deny introduced in [1] and [2] the notions of regular functional spaces and Dirichlet spaces. They treatised potentials in such a space. In potential theory, their method is very important for the research of the kernels satisfying the domination principle or the complete maximum principle. But the notion of a regular functional space is not sufficient, because the kernel of a regular functional space is symmetric if it exists. In this note, we shall extend the notion of a regular functional space and show the similar results as Beurling and Deny's. The detail will be published later elsewhere.

2. Let X be a locally compact Hausdorff space where there exists a positive measure ξ satisfying $\xi(\omega)>0$ for any non-empty open set ω in X, and let $C_K=C_K(X)$ be the space of finite continuous functions defined in X with compact support provided with the usual topology.[1] We define an extended regular functional space with respect to X and ξ as follows:

Definition 1. A Banach space $\mathfrak{X}=\mathfrak{X}(X;\xi)$ is called an extended regular functional space (with respect to X and ξ) if each element of $\mathfrak{X}$ is a real-valued locally ξ-summable function defined almost everywhere for ξ simply, *a.e.* in X and the following three conditions are satisfied:

(1.1) For each compact set K in X, there exists a positive constant $A(K)$ such that

$$\int |u(x)|\, d\xi(x) \leq A(K)\, ||u||$$

for any u in $\mathfrak{X}$.

(1.2) The intersection $C_K \cap \mathfrak{X}$ is dense both in C_K and in $\mathfrak{X}$.

(1.3) There exists a continuous bilinear form $\alpha(\cdot,\cdot)$ on $\mathfrak{X}$ such that $\alpha(u,u)=||u||^2$ for any u in $\mathfrak{X}$.

In the above definition, the norm in $\mathfrak{X}$ is denoted by $||u||$. For example, we can construct an extended regular functional space for a uniformly elliptic differential operator of order 2 which is not

1) That is, the net $(f_\alpha)_{\alpha\in I}$ is called to converge f in C_K if there exists a compact set K in X such that the support of f_α is contained in K and (f_α) is uniformly convergent to f.