95 *A Note on the Analyticity in Time and the Unique Continuation Property for Solutions of Diffusion Equations*

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1. Introduction. Consider the equation of evolution in $L^2(G)$

(1) $$du/dt = Au, \qquad t>0,$$

where G is a domain in R^n. We assume that A is an infinitesimal generator of holomorphic semi-groups $S(t)$ with the domain $\boldsymbol{D}(A)$ of $A\supset C_0^\infty(G)$, and that $A\varphi=\sum_{i,j=1}^n a_{ij}(x)\partial^2\varphi/\partial x_i\partial x_j+\sum_{j=1}^n a_j(x)\partial\varphi/\partial x_j+a(x)\varphi(\equiv A\varphi)$ for $\varphi\in C_0^\infty(G)$ where the coefficients satisfy the following conditions: $a_{ij}(x)$ are functions of class C^2 and with second derivatives locally Hölder continuous, i.e., $a_{ij}(x)\in C_{loc}^{2+h}(G)(0<h<1)$, $a_j(x)$ are of C^1, and $a(x)$ of $C_{loc}^h(G)$; the matrix $\{a_{ij}(x)\}$ is positive definite everywhere in G. The purpose of this note is to show the following theorems.

Theorem 1. *For any $f\in L^2(G)$, there exists a function $u(x, t)$ in $C_{loc}^{2+h}(G\times(0, \infty)$ such that for any fixed $t>0$ $u(x, t)=S(t)f(x)$ after a correction of a null set of the space R^n. Moreover, for any fixed x in G, $u(x, t)$ is analytic in t.*

Theorem 2. *Let f be in $L^2(G)$. If for a fixed $t_0>0$, $S(t_0)f(x)=0$ for almost every x in some nonempty open subset U of G, then $S(t)f$ vanishes identically in $G\times(0, \infty)$.*

The regularity of semi-group solutions of the diffusion equations was studied by K. Yosida [1] H. Komatsu [2], and others, under somewhat strong conditions on the coefficients. The unique continuation property of solutions of the diffusion equations was studied by Itô-Yamabe [3], Mizohata [4], Yosida [1], Shirota [5], and others. The proof of Theorems 1 and 2, shown in the next section, is suggested by K. Yosida [1]. We can extend our results in some directions:

1°. Instead of Eq. (1), we can consider the equation $du/dt=A(t)u$, where $A(t)$ are generators of holomorphic semi-groups satisfying certain conditions.

2°. The condition that the restriction of A on $C_0^\infty(G)$ is an elliptic operator of second order can be weakened to the following one; It is an elliptic operator of order 2 m with smooth coefficients