94. On Integers Expressible as a Sum of Two Powers. II

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1. In a recent paper [2] we proved the following results:

Theorem 1. *There is n_0 such that for every $n \geqq n_0$ there are positive integers x and y satisfying*

$$n < x^h + y^h < n + cn^a,$$

where h is any integer $\geqq 2$,

$$a = \left(1 - \frac{1}{h}\right)^2 \quad \textit{and} \quad c = h^{2-(1/h)}.$$

Theorem 2. *For any $\varepsilon > 0$, there is $n_0 = n_0(\varepsilon)$ such that for every $n \geqq n_0$ there are positive integers x and y satisfying*

$$n < x^f + y^h < n + (c + \varepsilon)n^a,$$

where f and h are any integers $\geqq 2$,

$$a = \left(1 - \frac{1}{f}\right)\left(1 - \frac{1}{h}\right) \quad \textit{and} \quad c = hf^{1-(1/h)}.$$

The case $h = 2$ of Theorem 1 and the case $f = h > 2$ of Theorem 2 are due to Uchiyama [3], while the case $f = h = 2$ of Theorem 2 is due to Bambah and Chowla [1].

As pointed out in Remark 4 of [2] we can replace c, in Theorem 2, by $C = fh^{1-(1/f)}$; but the theorem with c is the better result if $f > h$.

In this note we obtain the following refinement of Theorem 2 and generalization of Theorem 1:

Theorem 3. *There is n_0 such that for every $n \geqq n_0$ there are positive integers x and y satisfying*

$$n < x^f + y^h < n + cn^a,$$

where f and h are any integers such that $f \geqq h \geqq 2$,

$$a = \left(1 - \frac{1}{f}\right)\left(1 - \frac{1}{h}\right) \quad \textit{and} \quad c = hf^{1-(1/h)}.$$

This follows from the case $h = 2$ of Theorem 1 and

Lemma 1. *Theorem 3 is true for $f > 2$.*

The proof of this lemma has similarities with, but is more complicated than, the proofs of Theorems 1 and 2 and their special cases in [1], [2], and [3].

2. *Proof of Lemma 1.* We write $[t]$ for the greatest integer $\leqq t$.